Optimum limit design for reinforced-concrete continuous beams

by

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Dr Eng. V. Petcu (Head, Plastic Analysis Laboratory, Building Research Institute, INCERC, Bucharest) wrote that the problem of elaborating a theory of the optimum limit design for reinforced concrete structures had arisen from the fact that (1) particular ultimate and working conditions governed the limit design of these structures, and (2) variation in the percentage of steel in any critical section was always possible. This problem was of great importance not only from the theoretical point of view but for practical purposes as well. Therefore, the Paper was of a special interest for both research and for engineers concerned with reinforced concrete design.

90. Since 1961, considerable work had been done in Rumania concerning the elaboration of a theory of the optimum limit design for reinforced concrete structures. These might be obtained by two methods, which the writer had called the 'direct' and 'indirect' methods, respectively.¹⁴

91. The first consisted in determining the design plastic moments of structures by increasing (if necessary) some or all the plastic moments obtained in the manner presented below, assuming a full redistribution of internal forces under permanent and ultimate live load, so that working conditions and rotation compatibility of plastic hinges was fulfilled. The increase of the plastic moments was made by increasing the part of these moments corresponding to the live load, exclusively.

92. The indirect method consisted in decreasing the maximum elastic moments obtained under the assumption of a full elastic behaviour of structures under permanent and ultimate live loads, so that the design plastic moments resulting in this way fulfilled all the working and ultimate conditions. The decrease of the elastic moments was made by decreasing the part of these moments corresponding to the live load exclusively.

93. Both the direct and the indirect methods were based on the concept of the redistribution factor. As had been stated elsewhere¹⁵ the redistribution factor conceived as the support of the optimum limit design theory of reinforced concrete structures represented a general and unitary concept since: (1) it is a rational base for developing a general theory of the optimum design of reinforced concrete structures; (2) it represents a quantitative evaluation of the plastic adaptability of structures; (3) its expressions contains the yield load factor in an explicit form (see equation 48).

94. The last statement showed that the yield load factor was part of the redistribution factor. Therefore the theory of the optimum design based on the yield load factor⁶ represented only a variant of the theory based on the redistribution factor.^{16, 17} This conclusion was also underlined by the fact that the results obtained according to the first optimum limit design procedure presented by the Author coincided with those already obtained earlier by the writer.^{18, 19, 20}

95. The yield load factor did not represent the only variant of the redistribution factor concept. For example, in a previous paper²¹ another variant, called by the writer the 'plastic overload factor concept' was presented. In this the plastic overload factor was defined as the ratio between the ultimate live load and the live load causing the first plastic hinge.

96. Both the yield load factor and the plastic overload factor were of practical importance only for structures made of linear members.^{15,21} For all other structures, such as slabs or floors, the simplest way to obtain the optimum solution was based on the redistribution factor.²²

97. Referring to the direct method, the full redistribution of internal forces, which represented its starting point, was characterized in theory by an infinite number of solutions of plastic moments, among which there was the so-called optimum solution. The principle representing the theoretical base of the optimum solution, called by the writer 'the optimum redistribution principle', was firstly stated in 1961.^{16,17} The basic idea was that the best limit criterion was to ensure the same ultimate safety for all possible elementary mechanisms, the ultimate safety of the remaining elementary mechanisms being greater than the first.

98. The theoretical aspects of the optimum redistribution principle based on the redistribution factor concept had been presented in 1961 and applied to reinforced concrete continuous beams since then.^{16,17} A variant had subsequently been worked out by the Author in terms of the yield load factor⁵ and presented in the Paper in an abridged form.

99. Following the optimum redistribution principle, the writer had already obtained optimum solutions including plastic moments, redistribution factors, and yield load factors for many types of continuous beams subjected to different types of permanent and live loads.^{18-20, 23-26}

100. Referring to the indirect method, the decrease of the elastic moments was initially worked out by the writer taking into account a single value of the redistribution factor for all critical sections.¹⁴ This method was further generalized for cases where a value of the redistribution factor was assumed for each critical section²⁷ and applied to the determination of the plastic moments of reinforced concrete continuous beams where plastic hinges in the spans did not occur before the live load reached its ultimate value.²⁸ The main working limitation affecting both the direct and indirect methods was that plastic hinges did not occur under working loads.^{29,30}

101. As a result of the studies performed in Rumania, practical rules for the plastic design of reinforced concrete continuous beams with equal spans, including rotation compatibility of plastic hinges, were elaborated in 1963³¹ and typical illustrative examples were also given.³²

102. The above considerations showed the coincidence between the essence of the two optimum limit design procedures presented in the Paper and the direct and indirect methods of the writer. It was also worth mentioning that in the theory of the optimum limit design of reinforced concrete structures both the concept of the yield load factor and the concept of the plastic overload factor were derived from the more general concept of the redistribution factor.

Mr A. M. N. Amarakone (Research Assistant, Dept. of Civil Engineering, Imperial College, London) wrote that the technique of ultimate load design applicable to statically indeterminate structures as opposed to the design of sections as permitted by the present CP 114 had been the subject of intensive investigations in the recent past.³³ The need for an evaluation of the stress and deformation states of the structure under both working load and ultimate load had been emphasized from early attempts to formulate the basis of ultimate load methods.³⁴ In the Paper under discussion an attempt to rationalize the structural analysis to incorporate serviceability and the overload factors by the introduction of a yield safety parameter (y.s.p.) had been made. It was commendable in that it formed one of the first attempts to simplify design method by the introduction of limited redistributions based on a more rational analysis than had hitherto been possible.

104. In continuous beams (Fig. 1) it could be seen that the collapse mechanisms were relatively simple, being entirely independent, as each mechanism was localized to individual spans. Thus the provision of equal ultimate load factors could be

easily satisfied by the consideration of the equilibrium of separate spans as expressed by equation (13). However the possibility of excessive shear in conjunction with plastic hinges at ultimate load, and the limitation on the inelastic rotation capacity of the hinge sections which was needed in order to permit the necessary redistribution without any loss of the moment capacity of the hinges themselves, needed careful investigation.

105. The upper limit of 2% tension reinforcement (for mild steel reinforcement of 40 000/sq. in.) as a provisional recommendation appeared reasonable from the study of simple beam tests carried out under the European Concrete Committee.³⁵ The redistribution might however be further aided by the load requirements that at least one third of the area of bottom reinforcement should be provided as continuity bars over the support sections where the inelastic rotation was most critical. This steel, which acted as compression reinforcement, increased the rotation capacity of the plastic hinges. However if larger percentages of reinforcement were to be used, as in shallow beams, the redistribution at ultimate load could well be restricted by the actual percentage of tension reinforcement and the effective binding of the hinge sections. The distribution of the load itself might be an important factor.

106. The Author did not explicitly mention any criteria for the serviceability limit of the structure. In reinforced concrete structures designed for ultimate load with an allowance for large redistributions, some of the causes of unserviceability may be given by: (1) excessive working load stress; (2) excessive crack width at working load; (3) excessive deflexion.

107. The possibility of redistribution of up to 30% in some cases and the possible reduction of the overall load factor would actually increase the critical working load stress to very much higher values than those permitted in present design practice. Thus the unserviceability conditions might appear to be limitations on the redistribution, or might impose restrictions on the detailing of the critical sections.

108. The choice of a suitable yield safety parameter by the Author's method, apart from being an ideal optimizing criteria, was not explicitly related to the serviceability limit. The permissible upper limit on working stress depended on the nature of the live load and could well lead to a larger y.s.p. than the optimum value arrived at by this method. In any case it would be desirable to relate the y.s.p. to the actual requirements for serviceability.

109. The recommendation of a minimum percentage of 0.5% against excessive crack width seemed inadequate, as the crack width depended on many parameters, the most important being the ratio of tension reinforcement to diameter of bars, the concrete cover, and the maximum stress in the tension steel.³⁶ Although the calculations of crack width might be approximate, recommendations of the European Committee might be used to establish further limits on the detailing of the critical sections, depending on the type of structure and the crack width limit itself. As an example it could be shown that the maximum ratio of working load moment to yield moment at any section, based on a limit crack width Δl could be given by

$$\frac{M_1}{x_1} = \frac{\Delta l}{A} \frac{\mu'}{\phi} f_s \quad \text{where}$$

ϕ = diameter of tension bars

μ' = percentage of tension reinforcement based on equivalent tie bar

f_s = yield stress in tension reinforcement

A = an empirical constant.

Thus the minimum safety factors against first yield might lead to lower working load stress requirements than those assumed by the Author.

110. The limit on deflexions that could be permitted in common structures were not quite significant in the present design methods which conformed to elastic analysis and relatively small redistributions, but provision might have to be made to account

for increased deflexions due to reduced flexural stiffness of the beams as a result of the higher stresses allowed for in the limit design method.

Mr E. Burnett (M. S. Yolles Associates Ltd) congratulated the Author on this Paper, which surely represented a significant contribution to the development of a rationale for the design of reinforced-concrete structures. His own contribution concerned (1) ideal optimum criteria; (2) yield safety parameter; (3) dead load factor.

112. The general limit and serviceability criteria for continuous beams were given in § 17. As they stood these criteria had no qualitative significance. The limit criterion presupposed some chosen value for the ultimate safety and, while this occurred in practice, it could hardly be called the ideal optimum criterion. Possibly a more comprehensive limit criterion would be for all the elementary mechanisms to have the same maximum ultimate safety. Similarly, the ideal serviceability criterion would be for all critical sections to have the same minimum yield safety. The first OLD procedure would then derive directly from these criteria and, as such, would be theoretically a true optimum solution.

113. Although the scope of the Paper was limited to continuous beams of equal span and fairly simple loading systems, it was of interest to consider the yield safety parameter in a more general sense, though still limited to systems which consisted of elementary beam mechanisms only. $x_1 = c_{11}/c$ where for a certain factored loading condition c_{11} pertained to a particular critical section and c to the resulting beam mechanism. In theory it was possible to have s values for c_{11} (i.e. s loading schemes) and m different values for c . The yield safety parameter was thus a fairly complex function. In the derivation of OLD procedures it was possibly more convenient to work in terms of the two load factors c_{11} and c . The yield safety parameter and the redistribution factor were undoubtedly very significant parameters, but in the writer's experience^{37,38}, the separation of the load factors did permit some simplification.

114. The first OLD procedure would then reduce to the derivation of equal minimum values for c_{11} with c constant for all elementary mechanisms. In the development of the second OLD procedure, the criteria were given in § 46 as:

Limit criterion: a minimum ultimate load factor be ensured against each possible mechanism, but not necessarily the same for all mechanisms.

Serviceability criterion: equal yield load factors for all supports, equal yield load factors for all span critical sections with larger yield safety for spans than for supports.

Equations 33 and 34 remained unchanged, except that x_1 might be written as c_{11}/c where c might be different for each mechanism.

115. The service conditions should, however, be written as follows:

$$\begin{aligned} c_{10} &= c_{12} = c_{14} \dots c_{1, 2j} \\ c_{11} &= c_{13} = c_{15} \dots c_{1, 2j+1} \\ c_{10} &< c_{11} \\ \text{and} \quad c_{10} &> 1 \end{aligned}$$

where
and

With m possible values for c , all greater than or equal to the minimum, it did not follow that

$$\begin{aligned} x_0 &= x_2 = \dots x_{2j} \\ x_1 &= x_3 = \dots x_{2j+1} \\ x_0 &< x_1 \end{aligned}$$

nor that
nor that

116. It was current North American practice to use load factors for both the dead and the live loads. In general, the dead load factor was significant and the dead load effects were neither negligible nor dominant in the design of the structure. In spite of its logic it was likely to be some time before the use of an unfactored dead load was permitted. Would the Author please comment on adapting o.l.d. procedures to incorporate this and what effect it would have on the design recommendations in § 55.

The Author appreciated the contributions to the discussion of the Paper, and particularly those of Dr Petcu, which offered an account of the outstanding work in the field carried out in Rumania.

118. In §§ 94 and 102 the choice of the 'redistribution factor' in preference to the 'yield load factor' (or 'plastic overload factor') was discussed. The Author first introduced the concept of the redistribution factor in 1956,³⁹ and had pointed out the variables affecting the redistribution factor and its significance for the serviceability of structures under proportional loading:⁴⁰ no plastic hinges form at working loads as long as the redistribution factor does not exceed the specified load factor,⁴¹ i.e. if $r < c$. It was the merit of Dr Petcu's work that he had extended the use of the redistribution factor to combined loads and had applied this concept to the design of reinforced concrete continuous beams.

119. However, it was the Author's opinion^{5, 42, 43} that the redistribution factor only enabled a particular formulation of the serviceability criteria, in fact being neither indispensable nor the most convenient method of deriving OLD solutions. The essential point in these solutions was the adoption of specific design criteria, and not the concepts chosen for expressing these criteria.

120. As to the 'plastic overload concept' (ρ), as defined by Dr Petcu this seemed to be merely the inverse of the Author's y.s.p. ($\rho = P_u/P_1 = 1/x$), and it appeared to complicate the already rather complex terminology of limit design.

121. In § 96 Dr Petcu has suggested that the use of the x (or ρ) concept was 'of practical importance only for structures made of linear members'. In fact, in slab design, since proportional loads were postulated, the relationship between the three factors under discussion could be simplified to $r = \rho = 1/x$. If cM were the elastic moments for the ultimate loads, the design plastic moments for the ultimate loads were evidently $\bar{X} = cM/r = cM/\rho = xcM$. Hence the use of the yield safety parameter was as efficient in slab as in beam design.

122. The Author had no particular preference for either the redistribution factor or the yield safety parameter. It was his personal opinion, however, that the use of x in OLD was more straightforward, especially for proportional loads. In this case the y.s.p. became a correction factor of the elastic moments, an approach which was very familiar to civil engineers and common to many codes of practice.

123. The theoretical basis of Dr Petcu's methods (§ 97) was the 'optimum redistribution principle'^{16, 17}. The Author had pointed out the limit and serviceability criteria implicit to this principle along with its arbitrary nature and inherent limitations.^{5, 42}

124. The following remarks on Dr Petcu's methods might be made.

- (1) No reason was offered as to *why* the optimum solution was obtained by the principle postulated.
- (2) Even the 'indirect' method could not be explained by the optimum redistribution principle. Apparently this method (§ 100) called for minimum r (or $r = 1$) at all, or span critical sections, whereas the optimum redistribution principle implied maximum r for the majority of critical sections).
- (3) The principle did not seem to be as general in its applications as was suggested: it could not be applied to collapse in mechanisms other than of beam type. Indeed solutions based on $r_0 = \text{maximum } r$, as required by the principle, would result in collapse mechanisms at load factors less than those prescribed, and would therefore be unsafe.
- (4) The principle did not apply to beam mechanisms, unless fixed loading schemes were involved. For moving loads other criteria became mandatory.⁴⁴

125. The OLD methods presented in the Paper were based on the general principles of limit analysis,⁴⁵ supplemented by some convenient working conditions. Those essence could be expressed as follows.

- (1) Plastic moments were taken as equivalent to some reduced elastic envelope moments.
- (2) Specific design (limit and serviceability) criteria were adopted.
- (3) Design criteria were varied depending on the structural configuration and loading conditions.⁴³

As a result a whole spectrum of design criteria could be considered, of which those suggested by Dr Petcu's principle were only one particular choice. A variety of alternative solutions could therefore be produced, each being optimal in regard to the accepted criteria.

126. On this basis the two OLD methods in the Paper had been derived and unified solutions obtained for continuous beams with *any* type of static loading, various support conditions, and various numbers of spans. Similarly, it was possible to extend the application of OLD to moving loads,⁴⁴ to concrete frames,⁴⁵ or to other structures.

127. Furthermore, an absolute optimum solution, in the mathematical sense, would also be possible by postulating an objective optimum criterion (as for example, minimum cost, minimum material consumption, etc).

128. Commenting on the significance of the Author's criteria (§ 112), Mr Burnett had suggested statements almost identical to those in the Paper (§ 17). However, his only alterations (the 'maximum' and 'minimum' stipulations in the two design criteria) should be corrected to 'specified' and 'maximum' respectively. Indeed the same yield safety parameters could be assigned to all critical sections only if they had a common *maximum* (and not minimum) value. Otherwise, for any $\bar{x} < \max \bar{x}_k$ the lower bound theorem of plastic analysis was violated, i.e. possible collapse mechanisms would occur at load factors less than the specified c . On the other hand it was pointed out (§ 18) that the above criteria could be used simultaneously to yield a unique solution only in special cases, as in general they led to a number of conditions in excess of that of the unknowns.

129. Mr Burnett had noted a possible inconsistency in expressing conditions (35), (36), and (37). This was probably due to a misunderstanding of the definition of c , which in the Paper was taken as the 'specified (standard) value of the ultimate load factor'. By specifying a standard load factor for a structure, all c_{11} were divided by a constant c value and the conditions in terms of $x_1 = c_{11}/c$ were therefore correct. The limit criterion in § 46 was clarified in the discussion on the actual ultimate safety in §§ 62 and 63; the c_k and not the c values are variables in the various possible mechanisms.

130. Mr Burnett had asked if OLD was applicable to an ultimate load definition other than the one adopted in the Paper. It had been shown⁴³ that the optimum solutions were independent of the basic assumptions adopted, in the sense that a particular solution corresponded to a selected set of assumptions. Beside the idealization of the design plastic moment and the material behaviour, the definitions of (1) the ultimate load, (2) the yield load, and (3) the loading history from Q_{11} to Q_u were required.

131. Now if, as suggested by Mr Burnett, the ultimate load was defined by $Q_u = c_D G + c_L P$ and in addition the first plastic hinge load was $Q_1 = c_{1D} G + c_{1L} P$ (c_D , c_L , and c_{1D} , c_{1L} being the ultimate and yield loads factors for dead and live loads respectively), two assumptions on the loading history could be made:

- (a) the full dead load was reached first and the live load increased proportionally afterwards (if $c_D G = G'$, since $c_{1D} = c_D$ the ultimate and yield loads become $Q_u = G + c_L P$ and $Q_1 = G' + c_{1L} P$ respectively, i.e. the same definitions as in the Paper);
- (b) the dead and live loads increased at the same rate, i.e. $c_D/c_{1D} = c_L/c_{1L}$.

In the first case the recommendations in § 55 would apply. In the second case the optimum solutions could be obtained by using $X = x_i c M_i$ in the limit equilibrium

equations and applying the OLD principles. These solutions were a function of $w = G/P$ and the recommendations in § 55 would have to be amended accordingly. With the load factors in the ACI Code 318-63 substantial reductions of elastic (u.s.d.) moments could still be achieved for $w = G/P < 1$.

132. The Author fully agreed with Mr Amarakone's remarks on the compatibility condition and the role of compression reinforcement. He was also correct in his outline of the service requirements (§ 105) to which the condition of elastic behaviour (absence of plastic hinges) at working loads should be added.

133. Though the relationship between these requirements and the y.s.p. values adopted in OLD might not be apparent it could be shown that such a direct relationship did exist for serviceability³⁰ and for compatibility:¹² the higher the value of x adopted the smaller were the crack widths, the working stresses, and the inelastic rotations. Mr Amarakone was also right in referring (§ 109) to the recommendation of a minimum 0.5% reinforcement, the formulation of which should be refined. The intention was not to eliminate further cracking checks, but merely to indicate an absolute minimum steel ratio below which members behaved as though they were formed of plain concrete.

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