Lag curves for flood routing through a reservoir

by

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Mr I. F. Christie (Lecturer in Engineering, University of Edinburgh) wrote that the Author's graphs would be useful for obtaining a first approximation to the reduction in a flood peak due to reservoir storage, provided the simplifying assumptions on which the curves were based were not overlooked.

24. In a recent paper,⁵ a description was given of the use of a general-purpose electronic analogue computer for reservoir lag problems. For purposes of comparison, this method had been used to solve the Author's example and the result was given in Fig. 3. The inflow and outflow hydrographs and the reservoir level-time curve reproduced in that figure were all drawn automatically, on a 14 in. $\times$ 10 in. sheet of graph paper, by the pen recorder attached to the computer. The Author's step-by-step numerical solution and the solution by a graphical method⁶ were in close agreement with the analogue computer solution. For this type of problem, the use of an analogue computer was advantageous for investigating a number of flood hydrographs or various possible weir lengths.

25. In the Author's example, the reservoir level was initially equal to the overflow weir level, the surface area of the reservoir was constant, the discharge over the

FIG. 3: SOLUTION OF AUTHOR'S EXAMPLE OBTAINED FROM AN ANALOGUE COMPUTER

overflow weir was proportional to $h^{1.5}$ and the inflow hydrograph was of a special shape. None of these restrictions needed to be applied if an analogue computer was used.

26. Using the Author's notation, the equation to be solved was

$$a \frac{dh}{dt} = q_1 - q_2.$$

In general, $q_1 = f_1(t)$, $q_2 = f_2(h)$ and $1/a = f_3(h)$. A block diagram indicating how this might be represented on an analogue computer was shown in Fig. 4.

27. For special cases, such as the Author's example, some of the function generators might be replaced by simpler components such as amplifiers and potentiometers.

FIG. 4: CIRCUIT DIAGRAM FOR SOLUTION OF RESERVOIR LAG PROBLEMS BY ANALOGUE COMPUTER

Mr J. L. Milverton (Engineer, Freeman, Fox and Partners) wrote that his method was the determination of the time taken for successive increases of reservoir level above the spillway. Allowances could be made for variations of reservoir area, inflow, and discharge coefficient of the spillway, and the time for each increase was directly determined.

29. The basic assumption was that when the level above the spillway changed between two values, the rate of change of level was linear. By taking small changes of level, the error involved in this assumption was unimportant.

30. The spillway discharge at height h is $Q = Kh^{3/2}$, where K was effective length of the spillway $\times$ discharge coefficient. The area of this curve between limits $h=a$, and $h=b$ was by integration: $\frac{2}{5} K (b^{5/2} - a^{5/2})$. Therefore, the average value of Q between a and b was KR where $R = \frac{0.4(b^{5/2} - a^{5/2})}{b - a}$.

31. Values of R were given in the following table, for intervals of 0.5 ft and 1 ft, and were also plotted in Fig. 5.

Height above spillway <i>b</i> ft	<i>b</i> - <i>a</i> = 0.5 ft	<i>R</i>	<i>b</i> - <i>a</i> = 1 ft
0	0		0
0.5	0.14		
1.0	0.66		0.40
1.5	1.41		
2.0	2.32		1.90
2.5	3.38		
3.0	4.57		3.94
3.5	5.84		
4.0	7.28		6.56
4.5	8.80		
5.0	10.32		9.56
5.5	12.08		
6.0	13.76		12.92
6.5	15.68		
7.0	17.60		16.64
7.5	19.36		
8.0	21.60		20.48
8.5	23.6		
9.0	26.0		24.8
9.5	28.0		
10.0	30.4		29.2

32. Average discharges for smaller intervals than 0.5 ft could also be found. If the average discharge was required between b ft and $b + x$ ft, when x was less than 0.5 ft, the value of R was taken for $(b + 0.4x + 0.3)$ ft.

33. The inflow could be taken as the form $B + St$ cusecs where B was the initial flow and S was the rate of increase in cusecs/min.

34. The average reservoir area for each successive rise was A_1, A_2 , etc. in square feet.

35. Then, considering the first 0.5 ft rise, the time taken being t_1 min, and the initial flow B_1

$$0.5 A_1 = 60 t_1 \left(B_1 + \frac{St_1}{2} - RK \right).$$

R from the graph was 0.16, so t_1 could be directly obtained. For the next 0.5 ft rise, i.e. from 0.5 to 1.0 ft B_2 would be $B_1 + St_1$, so $0.5 A_2 = 60 t_2 \left(B_2 + \frac{St_2}{2} - 0.6 K \right)$.

36. This process was repeated, altering the values of S, A and K as required. The procedure when the inflow changed was shown in the following example:

Assume the following data:

$K = 600 H^{1\frac{1}{2}}$

Average area of reservoir for first 6 in. rise was 4 000 000 sq. ft, increasing by 50 000 sq. ft for each succeeding 6 in. rise.

Flood hydrograph was:—

Stage (a) 400–2400 cusecs in 80 min, i.e. $S = 25$ cusecs/min.

„ (b) 2400 cusecs for 50 min.

„ (c) Reducing at rate of 10 cusecs/min.

Total time from start = T min.

Discharge at end of each rise = Q cusecs.

FIG. 5: DETERMINATION OF R

FIG. 6: RESERVOIR LAG—COMPARISON OF METHODS

0-0.5 ft

Equation was

$$\frac{0.5 \times 4\,000\,000}{60} = t_1 \left(400 + \frac{25}{2} t_1 - 0.14 \times 600 \right)$$

i.e. $t_1 = 41$ min. $T = 41$ min. $Q = 212$ cusecs.

0.5-1.0 ft

$$B_2 = 400 + 41 \times 25 = 1425 \text{ cusecs.}$$

Therefore, $\frac{0.5 \times 4\,050\,000}{60} = t_2 (1425 + 12.5 t_2 - 0.66 \times 600)$

i.e. $t_2 = 25$ min. $T = 66$ min. $Q = 600$ cusecs.

1.0-1.5 ft

$$B_3 = 400 + 66 \times 25 = 2050 \text{ cusecs.}$$

Only 14 minutes of stage (a) remained, so this step would go into stage (b). Average inflow for remainder of stage (a) was 2225 cusecs.

Therefore, $\frac{0.5 \times 4\,100\,000}{60} = 2225 \times 14 + 2400 (t_3 - 14) - 1.44 \times 600 t_3$

$t_3 = 24$ min. $T = 90$ min. $Q = 1100$ cusecs.

1.5-2.0 ft

This was all in stage (b)

i.e. $\frac{0.5 \times 4\,150\,000}{60} = t_4 (2400 - 2.32 \times 600)$

$t_4 = 34$ min. $T = 124$ min. $Q = 1700$ cusecs.

Above 2.0 ft

Only 6 min of stage (b) remained, so it was unlikely that the next 0.5-ft step would be completed.

Try a rise of 0.20 ft.

Then the value of R was taken off the graph for $2.0 + 4 \times 0.2 + 0.3 = 2.38$ ft,

i.e., $R = 3.12$.

The equation was, therefore,

$$\frac{0.2 \times 4\,200\,000}{60} = 6 \times 2400 + (t_5 - 6)(2400 - 5t_5) - 3.12 \times 600 t_5.$$

This gave one value of t_5 of 53 min and, therefore, this represented the peak.

The maximum discharge was, therefore, $600 \times 2.2^{3/2}$ or 1960 cusecs.

at 53-6, i.e. 47 min. after start of stage (c).

The inflow then was $2400 - 47 \times 10$ i.e., 1930 cusecs.

If required, the process could be continued with a falling discharge.

37. A further example was given in Fig. 6, where this method was compared with the step-by-step calculations as shown in Table 2 of the Paper. The hydrograph had been simplified to straight lines and it was seen that the two outflows were very close. Taking 1-ft steps, only 10 steps were needed, and the process was rapid.

38. This method could be used in reverse, i.e. if the increase in height over the spillway was accurately logged, the flood hydrograph could be obtained.

39. This was done by assuming that the rate of increase of flow, S , was the same for the first two increments of height.

40. Then for the first 6-in. rise:

$$0.5 A_1 = 60 t_1 \left(B_1 + \frac{S t_1}{2} - 0.14 K \right)$$

41. This reduced to a simple equation for B_1 and S , as all the other factors were known.

42. For the second 6-in. rise,

$$B_2 = B_1 + St_1,$$

i.e.
$$0.5A_2 = 60 t_2 (B_1 + St_1 + \frac{St_1}{2} - 0.66K).$$

43. This, again, reduced to a simple equation for B_1 and S , which, together with the first equation, could be solved simultaneously, giving B_1 and S .

44. Then, for each succeeding increment, the new value of S could be obtained and the hydrograph determined.

Mr M. J. H. West (Executive Engineer, Water Development and Irrigation Division, Mwanza, Tanganyika) wrote that the Paper was of great interest, coming as it did from a country which must have many hydrological problems in common with Tanganyika. However, many of Tanganyika's dams—especially on smaller catchments—had 'natural' spillways in the form of a channel around one end of the dam, and, in those cases, the curves would not be applicable. Equation (9) was a simple expression which should give the peak outflow to within a few per cent of the actual.

46. The method was to assume a maximum permissible depth of flow over the spillway, find the corresponding attenuation of the peak of the design storm, and design the length and slope of the spillway to take the resulting peak outflow. Two approximations were necessary: first, that the inflow hydrograph could be approximated by a trapezium of base length T/h and crest length t/h (long recession

FIG. 7: RAPID APPROXIMATE GRAPHICAL SOLUTION TO AUTHOR'S EXAMPLE

curves should be disregarded when doing this, because they would usually not affect peak outflows); secondly, that the area under the rising limb of the outflow hydrograph was approximately equal to that under a straight line drawn from the beginning to peak outflow.

47. If V = capacity of reservoir between the spillway crest contour and the high flood level contour in acre-ft,

Q = maximum peak inflow in cusecs. Q_2 = maximum outflow in cusecs.
Then V equalled the area (shaded in Fig. 7) under the inflow hydrograph and above the outflow hydrograph divided by 12.

$$(1 \text{ acre-ft} = 12 \text{ s/ft h})$$

$$\text{That is } 12V = Q \left(\frac{T+t}{2} \right) - \left(Q_2 \times \frac{T}{2} \right)$$

$$\text{whence } Q_2 = Q - \frac{24V - Qt}{T} \quad . . . (9)$$

48. Applying this method to the example given in the Paper, and plotting the hydrograph given by Table 2 (see Fig. 7), with T converted to hours, the trapezoidal approximation had a crest of 1.7 h, and a base of 13.0 h.

If (for comparison) we assumed that the maximum flood lift would be 9.93 ft, then $V = \frac{9.93 \times 50\,000\,000}{43\,560} = 11\,400$ acre-ft.

(This figure could be found more accurately from the depth-capacity curve.)

$$\text{Then peak outflow} = 53\,500 - \frac{(24 \times 11\,400) - (53\,500 \times 1.7)}{13}$$

= 39 450 cusecs, compared with 41 200 cusecs by step-by-step method—an error of $4\frac{1}{4}\%$.

49. Applying this method to the example given by Richards,⁴ we have, for the given inflow hydrograph:

$Q = 253$ cusecs with initial flow of 25.3 cusecs

Rise in $W.L.$ = $1.98 - 0.48 = 1.5$ ft. Area = 50 acres

$V = 1.5 \times 50 = 75$ acre-ft.

$t = 4.5$ h.

T for main flood = 18.7 h.

$$\text{Peak outflow} = 253 - \frac{(24 \times 75 - 253 \times 4.5)}{18.7} = 211.6 \text{ cusecs,}$$

compared with 209 cusecs by step-by-step method—an error of $2\frac{1}{2}\%$.

50. It thus appeared that one could use this expression with reasonable accuracy, and, knowing T , t and V , the peak outflow could be found quickly and easily for any type of overflow spillway.

Mr A. R. Kopec (Engineer, Howard Humphreys & Sons, Consulting Engineers, Guma Dam, Sierra Leone) wrote that the following graphical method of determining the lag effect of routing a flood through a reservoir was submitted as an alternative solution to the problem given by the Author. The method had the advantages that it was generally applicable and might be used in conditions where there was base flow over the spillway. Moreover, it was not confined to any particular shape of outlet works and permitted variation of reservoir surface area to be taken into account.

FIG. 8: GRAPHICAL DERIVATION OF OUTFLOW HYDROGRAPH
Constant reservoir area, no base flow.

Notation

52. Let dt = time increment
 T = time from start of inflow
 I_1 and I_2 = inflow at the end of succeeding time intervals dt
 O_1 and O_2 = outflow at the end of succeeding time intervals dt
 Im = mean inflow in time interval dt
 Om = mean outflow in time interval dt
 dh = change in reservoir level in time interval dt
 H = head over spillway crest
 A = reservoir surface area

Assumption

$$53. \quad \left. \begin{aligned} \frac{I_1 + I_2}{2} &= Im \\ \frac{O_1 + O_2}{2} &= Om \end{aligned} \right\} \text{ at } T + \frac{dt}{2}$$

This was valid only when dt was small.

Construction

54. Plot the inflow hydrograph and the spillway discharge rating curve as shown in Fig. 8. Scale Q , which gave the values of inflow into the reservoir and the spillway capacity, was common to both curves. On scale A mark off the reservoir area at spillway crest level. The scales and units must be chosen so that:

$$\frac{Q}{H} = \frac{A}{T} \quad \dots \dots \dots (10)$$

55. The equation would be satisfied if the following units were used:

$$\begin{aligned} Q &= \text{cusecs} \\ H &= \text{ft} \\ A &= \text{sq. ft} \\ T &= \text{seconds} \end{aligned}$$

56. In the example chosen it was convenient to use the following scales:

$$\begin{aligned} Q &= 20\,000 \text{ cusecs to 1 in.} \\ T &= 20\,000 \text{ seconds to 1 in.} \\ H &= 4 \text{ ft to 1 in.} \end{aligned}$$

Substituting in eq. (10)

$$\text{Scale } A = \frac{QT}{H} = \frac{20\,000 \times 20\,000}{4} = 100 \times 10^6 \text{ sq. ft to 1 in.}$$

57. Choose a convenient time interval dt and mark off the value on scale T . Construct an equilateral triangle EFG as shown such that:

$$\text{Slope } EF = \text{Slope } FG = \frac{2A}{dt}$$

From O_1 draw a line O_1H_1 horizontally to meet the spillway rating curve at H_1 . From H_1 draw a line at slope $\frac{2A}{dt}$ to meet a horizontal line through Im at J , then reflect it at the same slope to intersect the spillway rating curve at H_2 . From H_2 project a horizontal line to intersect line GI_2 at O_2 which represented the discharge over the spillway at the end of time interval dt and at a height of H_2 above spillway crest.

58. Successive values of O were obtained by repeating this step and dt might be varied according to the accuracy required.

FIG. 9: GRAPHICAL DERIVATION OF OUTFLOW HYDROGRAPH
Variable reservoir area, residual discharge from antecedent flood

Mathematical analysis

59. In Fig. 8 it was required to prove that the hatched areas $I_1 I_2 O_2 O_1$ and $A dh$ were equal so as to satisfy the basic equation of reservoir lag, i.e.

$$(Im - Om)dt = A dh \quad \dots \dots \dots (11)$$

Now, by construction,

$$\text{Slope } J H_2 = \text{Slope } J H_1 = \text{Slope } EF$$

$$\text{or} \quad \frac{Im - O_2}{y} = \frac{Im - O_1}{x} = \frac{2A}{dt} \quad \dots \dots \dots (12)$$

$$\text{also} \quad y + x = dh$$

$$\text{therefore} \quad x = dh - y \quad \text{but} \quad y = (Im - O_2) \frac{dt}{2A}$$

$$\text{therefore} \quad x = dh - (Im - O_2) \frac{dt}{2A}$$

Substitute in equation (12)

$$\frac{Im - O_1}{dh - (Im - O_2) \frac{dt}{2A}} \frac{dt}{2A} = \frac{2A}{dt}$$

$$(Im - O_1) dt = 2A dh - (Im - O_2) dt$$

$$(Im - O_1 + Im - O_2) dt = 2A dh$$

$$\left(Im - \frac{O_1 + O_2}{2} \right) dt = A dh$$

$$(Im - Om) dt = A dh \quad \text{which is equation (11)}$$

Variable area

60. In some reservoirs the surface area might increase considerably as the water level rises above the spillway crest. As a result of the increased storage available, there would be a corresponding reduction in the outflow.

61. This variation might be taken into account as shown in Fig. 9 where the relationship between reservoir area and elevation above spillway crest was plotted in the lower left-hand quadrant. This curve was then used to make progressive adjustments in the slope of lines EF and FG as the level of the water rose over the spillway.

Spillway characteristics

62. In practice, the spillway discharge coefficient was seldom a constant and it might not be possible to express the spillway capacity in the terms of a simple discharge formula. The solution given here was applicable to any shape of spillway for which the head/discharge relationship might be expressed graphically.

Antecedent flood

63. When considering the effect of catastrophic or design floods, it was usual to assume that at the beginning of the inflow hydrograph there was already some discharge over the spillway resulting from an antecedent flood. In the example shown in Fig. 9 it had been assumed that antecedent inflow had ceased, but that there was a residual outflow when the design flood reached the reservoir.

Results

64. For the purpose of comparison, in Fig. 8 the same values were used for the inflow hydrograph, spillway rating and reservoir area as those used by the Author. The graphical solution gave:

$$\text{Max. outflow} = 41\,000 \text{ cusecs}$$

$$\text{Max. depth of outflow} = 9.98 \text{ ft}$$

which compared closely with the results obtained by the Author.

65. In Fig. 9 different assumptions had been made to illustrate the method of construction when dealing with a variable area and a residual flow and the two sets of results were not therefore comparable. Although the same inflow hydrograph was used, it had been assumed that the spillway had a curved crest designed for a head of 5 ft and that the discharge coefficient varied as shown by Creager, Justin and Hinds.⁷ It had also been assumed that the area of the reservoir increased uniformly from 50 to 60 million square ft over a range of 10 ft above crest level and that at the beginning of the flood there was a residual flow over the spillway of 5000 cusecs.

The results were:

$$\begin{aligned}\text{Max. outflow} &= 44\,400 \text{ cusecs} \\ \text{Max. depth of outflow} &= 8.78 \text{ ft}\end{aligned}$$

The Author, in reply said that when writing the Paper, he had hoped that it would provide a quick approximate solution to the reservoir lag problem.

67. In many parts of Africa the hydrological data was so meagre that the calculation of maximum flood was very difficult and the determination of the flood hydrograph a matter of intelligent guesswork. If it was accepted that an empirical average hydrograph such as U.S. Soil Conservation hydrograph,⁸ was as good a guess as any, then the resulting maximum outflow could be calculated rapidly with the aid of the curves shown in the paper. However, in the case of major dam sites where the hydrological data was sufficiently accurate to warrant a more detailed analysis to allow for varying surface area, initial base flow and complex spillway discharge formulae, then resort would have to be made either to an analogue computer as suggested by Mr Christie or to one of the conventional step-by-step methods such as that shown in Appendix II of the Paper and which Linsley, Kohler and Paulhus³ ascribed to Goodrich.⁶

68. Mr Milverton had given a step-by-step solution based on head intervals in place of the usual time intervals. Despite the simple arithmetic involved in each of these steps, the Author was not altogether convinced that this method was any quicker than conventional solutions.

69. Mr West had shown an interesting approximate solution in assuming a maximum depth of outflow at the beginning of his calculation. In dam design it was usually of economic advantage to keep the dam freeboard as low as possible and therefore the spillway length was made as long as the physical characteristics of the site would allow. The problem was then to determine the maximum depth of outflow through this fixed spillway length. However, Mr West's method gave a rapid solution in those cases where it was decided to design to a fixed maximum depth of outflow and where it was possible to vary the spillway length. Presumably when designing to a fixed spillway length, the maximum flood lift could be determined by successive approximations.

70. Mr Kopec had shown an ingenious graphical variation of the conventional step-by-step method. As the mathematical basis was fundamentally the same in both cases, it was a matter of personal preference whether this method was used or the semi-graphical type shown in Appendix II.

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