A prestressed-concrete foundation for a 200-MW steam turbo-alternator

by

Edward Allan Leonard Keith, B.Sc.(Eng.), A.M.I.C.E.

Mr F. A. Rae (Senior Engineer, Blyth & Blyth, Consulting Engineers, Edinburgh) questioned the validity of the theory that prestressing the columns of a portal would alter the natural frequency of vertical vibration to the extent suggested in §§ 24, 25. His objection was based on the fact that the resonant frequency of free vibration of a given system depended solely on the mass and on the stiffness of the supporting structure, so that in its simplest form

$$\text{Frequency} \propto \sqrt{\frac{\text{Stiffness}}{\text{Vibrating mass}}}$$

72. This could be re-arranged in the form given in § 24 of the Paper, but it must be remembered that d in the expression was merely replacing the terms inside the square root in the formula above and, therefore, must be taken as the deflexion due to the mass which was to take part in the vibration.

73. Prestressing the columns would affect the equilibrium position of the vibration system, just as a change in the earth's gravitational pull would do, but neither of these would have any significant effect on the vibration frequency.

74. Mr Rae agreed that the prestressing cables would alter the vertical stiffness of the columns slightly, but the effect could be obtained more cheaply with ordinary mild steel of the same total cross-section and, moreover, the additional creep in the concrete which resulted from a prestress load would be absent.

Professor R. G. Robertson (Professor of Civil Engineering, University of Cape Town) wrote that the statement in § 25 that prestressing would lower the frequency, and that this was due to the deflexion caused by the prestressing, was not in accordance with Rausch's equation in § 19. In this equation the deflexions were described as those due to *load* and therefore dependent on the elasticity of the members, and not to any prestress in them.

76. Similarly in § 7 it appeared that prestress was credited with adding vertical load, which did not appear to Prof. Robertson to be fundamentally valid, because mass and elasticity were the criteria involved.

77. It was possible that a prestressed member had a greater stiffness than a similar-sized member in normal reinforced concrete, but this would have the reverse effect, and would increase the frequency.

Dr H. G. Allen (Lecturer in Civil Engineering, Southampton University) wrote that he was particularly interested in the statement in §§ 24, 25 that the natural frequencies of a portal could be diminished by the application of prestress to the columns.

79. This statement was based on the formulae given by Rausch⁷ for vertical oscillation of the mid-point of the beam and for horizontal (sway) oscillation of the portal.

In the latter instance, Rausch's formula was

$$n_h \doteq \frac{300}{\sqrt{d_h}}$$

where n_h = natural frequency of horizontal (sway) oscillations, in cycles/minute.

d_h = horizontal displacement (cm) of beam which would be caused by a horizontal force W at the level of the beam

W = a force numerically equal to the weight supported by the beam (including the weight of the beam and of the upper parts of the columns).

80. This formula could be derived easily from the simple idealized system in Fig. 13, which represented a mass M supported by an elastic weightless cantilever of stiffness k .

FIG. 13

Stiffness here was defined as the horizontal force which must be applied at the tip of the cantilever in order to produce a unit horizontal displacement at the point of application.

81. The natural frequency of the system was

$$\frac{1}{2\pi} \cdot \sqrt{\frac{k}{M}} \text{ cycles/sec.}$$

If one wrote, as Rausch did,

$$d_h = \frac{M \cdot g}{k},$$

then k could be expressed in terms of d_h and the expression for natural frequency became

$$\frac{1}{2\pi} \sqrt{\frac{M \cdot g}{d_h} \cdot \frac{1}{M}} = \frac{1}{2\pi} \sqrt{\frac{g}{d_h}} \text{ cycles/sec.}$$

or

$$\frac{60\sqrt{g}}{2\pi} \cdot \frac{1}{\sqrt{d_h}} \text{ cycles/min.}$$

82. If d_h were expressed in centimetres, the substitution $g = 981 \text{ cm/sec}^2$ gave $\frac{300}{\sqrt{d_h}}$ (Rausch's formula). If d_h were expressed in inches, the substitution $g = 32.2 \times 12 \text{ inches/sec}^2$ gave $\frac{187}{\sqrt{d_h}}$.

83. The point of Dr Allen's comment was that the natural frequency was a function only of (1) the mass supported by the portal, and (2) the stiffness of the columns. Prestressing the columns did not increase the mass nor did it affect materially the bending stiffnesses of the columns. (The arguments of § 66 applied here also.) The deflexion

• A correction to this effect was noted in July 1962 Proceedings, p. 352.

d_h was merely an algebraic device used by Rausch to eliminate the stiffness and mass terms from the expression for natural frequency; it signified the lateral deflexion caused by a force equal to the weight of the mass resting on the portal. The prestressing force had no associated mass and therefore did not enter into the calculation of d_h .

84. Similar reasoning could be applied to the formula for vertical oscillations. In consequence it would seem that the application of prestress to the columns had no primary effect on the natural frequencies of the portals.

Professor M. R. Horne (Professor of Civil Engineering, University of Manchester) wrote that the central thesis of the Paper, namely that the natural frequencies of foundation blocks could be adjusted by the degree of prestressing, was untenable. It was derived from fallacious theoretical arguments, and was unsupported by experimental evidence.

86. In Rausch's formula for vertical oscillations $\left(n_v = \frac{300}{\sqrt{d_v}}\right)$, the vertical deflexion d_v represented the ratio of the supported mass M to the vertical stiffness λ of the beam or portal, and was derived from the usual formula for the frequency of a suspended mass $\left(n = \frac{1}{2\pi} \sqrt{\frac{\lambda g}{M}}\right)$ which, in c.g.s. units with $g = 980$ cm/sec², gave $n = 299 \sqrt{\frac{\lambda}{M}} = \frac{299}{\sqrt{d_v}}$ cycles/min). The Author had included in the quantity d_v the vertical deflexion due to prestress, and had thereby deduced that prestressing would reduce the natural frequency.

Stiffness of each spring = λ

FIG. 14: CONSTANCY OF NATURAL FREQUENCY WITH RESPECT TO VARIATION OF STATE OF INTERNAL STRESS

This entirely lost sight of the derivation of the formula. Actually, prestressing would increase stiffness to the extent that it prevented cracking under tension or the opening of shrinkage cracks already present, thus somewhat decreasing M/λ and increasing n_v . Provided sufficient prestress were present to prevent such cracking, additional prestress could have not the slightest effect on stiffness, and hence on frequency.

87. It was perhaps worth describing a simple model that would illustrate the fallacy. The rigid mass M in Fig. 14a was connected to three rigid foundations by three equal springs of stiffness λ as shown, the frequency for vertical oscillations being $\frac{1}{2\pi}\sqrt{\frac{3\lambda g}{M}}$. If the central foundation were given a downwards displacement (Fig. 14b), representing

the prestressing operation in the concrete blocks, it was evident that the natural frequency would remain unchanged. There was no difference in principle between the problem posed by this example and that of the prestressed structure under consideration.

88. The Author had used a similar argument for introducing an imaginary effect of prestress in the columns on the frequencies of horizontal oscillation. The idea that prestress could reduce these frequencies was again fallacious.

89. The Author had sought to transform Rausch's original formula, expressed with d in centimetres, into a formula with d in inches (§ 20). The correct transformation was

from $n = \frac{300}{\sqrt{d}}$ to $n = \frac{300}{\sqrt{2.54d}} = \frac{188}{\sqrt{d}}$ and not to $n = \frac{300\sqrt{2.54}}{\sqrt{d}} = \frac{478}{\sqrt{d}}$ as propounded. If the

error had indeed been carried through in the Author's calculations, the final corrected figures for frequencies would be somewhat below the final (i.e. lower) curves given in Fig. 8. This would mean that, by an extraordinary combination of mischances, the 200-megawatt foundation block, as designed, would be satisfactory on the basis of the assessment adopted by the Author.

90. There was a further major misconception contained in the Appendix (§§ 66–70) concerning the effect of prestressing on the oscillatory behaviour of the condenser beams. If the longitudinal prestressing force S acting on the ends of the beam (Fig. 12) was conceived as producing a bending moment Sy , then it was necessary to consider the additional effective distributed lateral load of intensity $S \frac{d^2y}{dx^2}$, produced by the curvature of the prestressing cables, except for the special case in which the cables were freely stretched in ducts of sufficient diameter for them to remain straight during the deformation of the beam. This case apart, the effects of the bending moment Sy and the lateral load $S \frac{d^2y}{dx^2}$ exactly and precisely cancelled. Prestress thus had no effect on the natural frequency of a beam except for its possible effect in preventing cracking, leading to a slight increase in frequency. The assertion that a prestressed beam would buckle if the prestressing force approached the Euler buckling load (§ 68) was entirely unfounded, and certainly untrue.

Mr R. G. Batson (Engineering Assistant, Rendel, Palmer & Tritton) said that the natural frequency of vibration of an undamped system could be determined from consideration of the balance of the inertia and restoring forces acting on the system. For a single mass system with one degree of freedom this could be written:

$$\frac{\omega}{g} \ddot{x} + kx = 0 \quad \dots \dots \dots (3)$$

where ω = weight of the mass, and k = system stiffness.

From this the fundamental natural frequency was given by

$$p = \sqrt{\frac{kg}{\omega}}$$

It was to be noted that the expression $\frac{\omega}{k}$ was the static deflexion of the mass and so the fundamental frequency could be written as

$$p = \sqrt{\frac{g}{d}} \quad \dots \dots \dots (4)$$

The equation could be used for more complicated systems, since the static deflexion was a function of the various masses and stiffnesses of the system.

92. Care had to be taken when using equation (4), for the reason which followed: Equation (3) showed that the inertia force was equal to the mass of the weight multiplied

by its instantaneous acceleration. Since both the displacement x and acceleration $\ddot{x}$ changed sign every half cycle, so the inertia and restoring forces had to, and hence the system oscillated.

93. An external force F , of constant sign and value, applied to the mass along the line of its oscillation, would not affect the frequency of vibration, but would simply move the centre of oscillation a distance $\bar{d}$ given by

$$\bar{d} = Fk.$$

This value $\bar{d}$ was not to be added to the static deflexion d in equation (4) to determine the fundamental frequency, since the force F which caused this displacement had no effect upon the inertia forces, because it was not associated with a mass, or upon the restoring forces, which had to change direction every half cycle, because it was constant in magnitude and direction.

94. In §§ 24, 25, the Author had indicated how he intended to decrease the natural frequency of vibration of the portals by prestressing the columns. For vertical oscillations, the prestressing force could be regarded as a force of almost constant magnitude and of constant direction, applied along the line of oscillation. Since this force had no effect upon the vertical stiffness of the columns and did not change sign every half cycle, it could have no effect upon the natural frequencies, but merely shifted the origin for vertical oscillations down by the amount of the elastic shortening of the columns.

95. The truth of this could very easily be seen. If the portal were to have been turned completely upside down, its vertical frequency would have been unchanged. In that case, however, the elastic shortening of the columns would need to have been deducted from the static deflexion of the beam centre, which according to the Author would then have raised the frequency!

Messrs B. Wood and L. Schlesinger (Head of Development Department, and Chief Civil Engineering Design Engineer, respectively, Merz and McLellan, Consulting Engineers) remarked on the seeming possibility that whereas foundations for turbo-alternators had been at one time successfully designed by rule of thumb and by experience, they might now be designed scientifically on lines which were incorrect theoretically. The gravamen of the Author's ideas of design, mentioned in § 6, was summed up in § 25 where he said that pre-stressing of the columns would lower the frequencies. This was untrue. Pre-stressing did not alter the deflexion of a member, either column or beam, caused by application of a given load. It also did not alter either Young's modulus or the density and hence did not alter the velocity of sound in the ferro-concrete, which, looking at the issue in another way, was what fixed the time of passage of an impulse and hence the longitudinal natural frequency in an unloaded column. It was true that pre-stressing could lead to smaller cross-sections, in which case the deflexion under a given static load would be increased and the natural frequency would be lowered, but the fact that the Author was not thinking of this was shown by his reference to 'tuning' by post-tensioning. Since, however, foundations of the classic type which had hitherto been used in England were always high-tuned, at least for the bearings supported directly by columns, such a change was undesirable as leading to a risk of resonance, which experience over the years had shown did not occur when foundations had been designed virtually by rule of thumb. Orthodox reinforced concrete achieved certain desirable properties by accident: The shrinkage cracks provided regions of effective if inadvertent damping; the large mass reduced amplitudes and increased stability. Provided the concrete was quality controlled to ensure that movement ceased after a few months, no harm resulted from hair cracks. One case was known to the writers where vibration had been bad until cracks had developed. There was a risk that closing up the micro-cracks by post-stressing might, by reducing the damping, cause troubles which had so far been avoided. Foundations possessed of high damping had an inherent design tolerance minimizing resonance troubles. This seemed to be the reason why,

although very widely differing types of foundation were in use in the U.K. and the U.S.A., none gave trouble except on rare occasions.

97. A full understanding of possible foundation behaviour involved greater complication than seemed to have been realized. The use of the fourth order differential equation as in the appendices § 48 and § 52 required over-simplification, since except for the simplest shapes, such as prisms, there was no means of inserting the boundary conditions. One was thus driven to the alternative Rayleigh method of trying to visualize each individual possible mode and calculating its frequency, which could be done by successive approximation if necessary.

98. The use of Rayleigh's principle allowed the expression attributed to Rausch in § 19 to be derived, though this could be done more simply from first principles. Taking the static deflexion curve as an approximation to the dynamic deflexion curve in the fundamental mode, and equating the maximum changes in kinetic and potential energy, gave

$$\sum \frac{1}{2} \cdot \frac{W}{g} \cdot \omega^2 d^2 = \sum \frac{W}{2} \cdot d,$$

where $\frac{W}{g}$ was the lumped mass of part of the structure and d the local deflexion under gravity by weights $\sum W$. If only one lumped mass was assumed, this simplified to the well-known form

$$\omega = 2\pi n = \sqrt{\frac{g}{d}},$$

or $n = \frac{187}{\sqrt{d}}$, with n cycles per minute and d in inches. § 25 suggested that the Author

would have inserted the elastic compression due to pre-stressing in this formula. This would have been incorrect as, first, the prestressing force did not produce a deflexion curve which was a good approximation to the dynamic deflexion, and, secondly, no account of the force had been taken in deriving the potential energy term. The strain energy induced by the pre-stressing operation was not converted into kinetic energy during vibration. Similarly, Fig. 8 suggested that the deflexion due to dynamic loading had been used, which was incorrect. Correct application of this formula would have raised the natural vertical frequency indicated, as would the use of a higher value of Young's modulus for concrete usually assumed for low-tuned foundations.

99. The principal effect of pre-stressing a concrete member would be to ensure that the full concrete section was effective, providing the pre-stress was sufficient to overcome tensile stresses induced by the dynamic loads.

100. The Author was trying to achieve a low-tuned foundation by means of pre-stressing. The large number of pre-stressing bars required, the use of high quality concrete, and the increased construction difficulties, all added to the cost. A structural steel foundation would have fulfilled his original object of a low-tuned foundation with greater certainty, much reduced cost, and saving in construction time. The chief merit of the low-tuned foundation, however, was the increase of available space under it for other equipment. This advantage would be somewhat dearly bought.

Mr R. Crowder (Design Engineer, Taylor Woodrow Construction, Ltd) observed that Mr Keith's Paper was extremely timely, there being in Britain little published work on foundations for vibrating machinery. The present trend, both in this country and abroad, was for mechanical engineers to produce larger and slower-running turbo-alternator sets, which in turn required larger foundation structures. It had been usual practice earlier, with smaller sets, to provide an adequate foundation of reinforced concrete which was then more or less expected to look after itself. Recently

it had become obvious that there were distinct economic and structural advantages in carrying out as thorough a vibration analysis of the foundation as possible.

102. The Author had set out to do just this, but it was felt that rather more emphasis could have been given to the design of the foundation raft, which after all constituted the major proportion of the concrete in the foundation. The raft depth of 20 ft was unduly great, and seemed to have been adopted solely to found the block on hard marl. Mr Crowder asked whether it might not have been more economical to have

Curves for Breadth/Depth Ratio 2:3

Simply supported Timoshenko curve from:

$$\frac{a^2 m^2 \pi^4}{l_0^4} - F_m^2 - \frac{F_m^2 m^2 \pi^2 r^2}{l_0^2} - \frac{F_m^2 m^2 \pi^2 r^2}{l_0^2} \frac{E}{KG} + \frac{r^2 \gamma}{gKN} F_m^4 = 0,$$

Where

$$a^2 = \frac{EI_g}{A\gamma} \quad r^2 = \frac{I}{a}$$

K = shape factor γ = density

FIG. 15: COMPARISON OF CURVES GIVING FACTOR FOR COMBINED FREQUENCY

piled down to this depth, or to have designed a raft founded on presumably softer and more elastic material overlying the marl, and to have provided heavier raft reinforcement. Current German practice was that the thickness of the raft should be about one tenth of the overall length of the turbo-block, which in this case would have given a not disproportionate depth of about 10 ft 6 in.; this should also have provided ample depth for anchoring the prestressing cables.

103. A point worth making was that the design of the raft should have been such that its centre of area coincided with the centre of gravity of the raft, superstructure, and machinery (in a horizontal plane). If this were so, the static settlement of the block would be uniform, as would be any settlement due to compaction under vibration; such a state of affairs was ideal as regards the alignment and smooth running of the machinery.

104. It was also felt that the Author gave little justification for prestressing the beams and columns of the superstructure, rather than using reinforced concrete. Certainly there was generally little benefit in reducing the section of the top beams and slabs, which were usually governed by a limiting deflexion for the turbine manufacturers. This lent itself to the design of these members as high-tuned (i.e. their natural frequencies were above the running speed of the machine). Furthermore, any reduction in the depth of these beams tended to enhance the problem of securing to them the usually massive turbines and generator. It could be inferred that prestressing guaranteed the utilization of the full concrete section, but this hardly seemed an economical way of doing it, since the deflexion of a high-tuned beam should be no more than 0.002 in.

105. The Author correctly pointed out in his conclusions the facility provided by prestressing for tuning the blocks. However, a little later he went on to say that no provision had been made to utilize this feature on the block in question, at High Marnham. Thus it could be concluded that it had been uneconomical to realign the machines after tuning, unless the limitations to tuning by prestressing were the crucial factors. Experience of the design of a foundation block for a 324.75 MW turbo-alternator set in reinforced concrete had yielded approximate reinforcement weights of 3 lb/cu. ft for the raft, 5 lb/cu. ft for the columns, and $4\frac{1}{2}$ lb/cu. ft for the beams.

106. Of particular interest was the Author's mention of the simplified vibration treatment of portals using Rausch's method. Various bodies of opinion felt that the resonance of individual beams might well be as important as that of portals considered as a whole. Difficulties were encountered in the assessment of the effective length and end fixity of the beams, which was overcome to a large extent by comparing the treatment of the beams as simply supported between vibration nodes over the clear span, and as fixed-ended over an effective length extending into the supports. Rausch's method had been found to give results varying from those derived from the more rigorous Timoshenko treatment which accounted for rotatory inertia as well as flexural and shear effects.

107. To facilitate quick vibration analysis, a series of curves had been evolved for given breadth/depth ratios of the beams (Fig. 15 showed the curves for a breadth/depth ratio of 2.3). For the known depth/span (d/l_0) ratio of the beam under analysis, a factor U could be selected by which the natural frequency of the beam under simple flexure was modified to account for shear and rotatory inertia. The curves gave the modified frequency F_m in terms of f_0 , the bending frequency, in the form $F_m = f_0 (1 + K d/l_0)^{-1/2}$, where K was a constant depending on end conditions.

108. The larger the unit of the block considered, the more involved and nebulous a rigorous vibration analysis would become. The factors of multiple coupling and damping which must effect the vibration of all members seemed to have much influence on portals and space frames usual in turbo-blocks. Here indeed was a subject which might merit intensive future research.

Mr R. H. Elvery (Lecturer, University College, London) pointed out that the resonance frequency of a structure of the type described in the Paper was a function of the stiffness of the members which made up the structure and was not dependent on the

stresses in the members. He was puzzled by the Author's statements in §§ 25, 42, which were contrary to this basic principle.

110. He was unable to agree with the suggestion that prestressing had the effect of reducing frequencies, since stress did not fundamentally reduce the stiffness of the structure and the mechanics of the system were unchanged. Several years earlier a research programme had been carried out at University College, London, to determine the effect of sustained stress on the dynamic modulus of elasticity of concrete.¹⁰ This had concerned a study of the changes in the elastic properties of concrete when under compressive stress. It was interesting to recall that these changes had been measured by observing the changes in the resonance frequency of concrete columns which had been prestressed with Macalloy bars. It had been expected that this prestressing would not cause any changes in frequency unless changes occurred in the properties of the concrete. The experimental results had shown this to be true and only small changes of frequency had been observed, these being due to the structural changes in the concrete. When prestress had been applied, the dynamic modulus had been reduced at first by up to about 4%, most of this being recovered within a month of stressing. A further reduction of modulus up to about 3% had occurred when the prestressing was removed. These small changes would have caused frequency changes of the order of about 2 or 3% in the columns and would hardly have been useful as a means of detuning the structure away from the alternator frequencies. These observations did not support Mr Keith's calculated results, given in Fig. 8, which showed that prestressing reduced the frequency by as much as 60 or 70%.

111. Mr Elvery was interested to know whether Mr Keith had carried out any resonance tests on the actual structure and whether any attempts had been made to alter its frequency by changes in the prestress, as suggested, although he did not expect that this would have been effective.

112. He was also interested to know what changes had been made in the calculations to produce the differences between unstressed and prestressed conditions as given by the graphs in Fig. 8.

Mr B. G. R. Holloway (Partner, Rendel, Palmer & Tritton) agreed that prestressing offered a number of advantages in the construction of turbo-blocks, but doubted whether it was possible to modify the frequency of vibration of concrete members by certain types of prestressing.

114. The frequency of a member could be changed during the prestressing operation, provided that the ducts for the tensioning cables were not grouted up afterwards. But with post-tensioned members where the cable ducts were properly grouted up, and with pre-tensioned members, the prestressing forces became internal forces and the stored energy from this source remained unchanged whatever the subsequent deflected shape.

115. The problem of stability and natural frequency of prestressed concrete members was dealt with in considerable detail by Paduart,¹¹ who stated that the natural frequency of a beam remained unaltered by prestressing when the ducts were fully grouted up.

116. This problem was analogous to the residual stresses in a rolled or welded steel beam or column where the natural frequency of the member remained unaffected.

Mr R. A. Waller (W. S. Atkins and Partners) noted that vibrating structures could be divided into two categories, those whose natural frequencies were affected by their state of internal stress, and those that were not. Unfortunately most well-known examples, such as violin strings, belonged to the first category. It was, perhaps, not surprising that in engineering circles there was a popular misconception that post-tensioning affected the natural frequency of a member. Contrary, therefore, to the Author's statement, post-tensioning did not *per se* alter the natural frequency of the foundation, as it did not directly alter either of the two variables involved, namely mass or stiffness.

118. Indirectly, post-tensioning had two effects on the stiffness. Elvery and Furst¹⁰

had found that compressive stress resulted in a reduction in the dynamic modulus of elasticity, but this effect was always less than 5%, and for the stresses in the present case the decrease would have been very small indeed.

119. The second and possibly somewhat larger effect was the increased stiffness due to the closing of shrinkage cracks in the concrete. Neither of these effects produced a change in stiffness large enough to rectify a resonant condition. Generally speaking, in a reinforced concrete structure, a change in stiffness of at least 20% was required for this purpose.

120. Mr Waller's firm had had to consider carefully the relative merits of reinforced concrete, prestressed concrete and steel in the construction of this type of foundation and of anti-vibration mountings. While not disputing the choice of prestressed concrete in this case, they had found that generally they preferred to use reinforced concrete. It had the advantage of damping characteristics much better than those of steel, and to some extent better than those of prestressed concrete. As structures of this nature tended to have a complicated geometry, it was always possible that some part of the structure would resonate. In that event, reinforced concrete tended to de-tune itself; by cracking it reduced the stiffness of the member and introduced greater damping into the system. Prestressed concrete would not behave in this way unless the vibration was so large that it overcame the precompression in the concrete. Normally, of course, stresses due to vibration were too low to do this.

121. The Author had raised the question of tuning the foundation subsequent to erection. It was important that this could be done when steel was used. In a case where speedy erection had been required, Mr Waller's firm had proposed to use a steel foundation made up of hollow box sections, incorporating on the inside of the members a number of shear connectors. In the event of an undesirable vibration one or more members could then have been filled with concrete.

122. With this type of foundation it was usual for the vertical natural frequency to exceed the running speed of the machine. The lateral frequencies, however, were nearly always well below the running speed and, as the Author had pointed out, it was important that the turbine did not linger at these speeds. This problem was even more acute in the case of variable-speed machines, for example carbon dioxide blowers in nuclear power stations. In such cases, as the lateral frequencies could not be raised above the maximum running speed, they had to be sufficiently low for the amplitude of movement during resonance not to result in large forces. One way of doing this was to interpose, in the columns of the foundation, laminated rubber bearings of the type now widely used as bridge expansion bearings. The top of the foundation would then be treated as a subframe which was sufficiently rigid in itself to maintain the alignment of the machinery. Vibration effects in the lower half of the foundations could be virtually ignored.

123. The idea that a foundation for this type of machinery must be massive and rigid was still widespread. This was due, Mr Waller thought, to a lack of fundamental understanding of vibrations by civil engineers generally, and perhaps to the idea that these foundations necessarily had to react large dynamic forces. In fact, most of the dynamic forces were a function of the stiffness of the foundation and, with natural frequencies below the running speed, the more flexible the foundation the less force it had to react.

Mr M. A. Aggett (Senior Engineer, Taylor Woodrow Construction, Ltd) observed that the 35 ft 6 in. clear span of the condenser beams for the High Marnham blocks must have approached the maximum at which it was economically possible to keep the dynamic deflexions within acceptable limits. If the turbine manufacturers had arranged the condenser layout differently and provided a central support to the condenser beams, it appeared that an overall economy might have been effected, not only by reducing the depth of the beams but also, in consequence, the height of the blocks and of the turbine hall.

125. One turbine manufacturer had provided an effective solution to the problem of condenser beams associated with large turbo-generators. The condenser structure was of the 'bridge' type, forming a deep girder which spanned the foundation superstructure between the steam and generator ends of the machine and was also supported by springs on the foundation raft in the usual way. This device had not only dispensed with the need for difficult concrete condenser beams but had also afforded an economy in the overall height of the block, a particular advantage where an underground basement was involved. This arrangement was being used at a nuclear power station then under construction, where the 325 MW capacity of each set was comparable with a 500 MW set for a conventional station where higher steam temperatures and pressures were used.

126. The steam chest and throttle valve cantilevers shown on Elevation P.P. (Fig. 2) were of unusually slender proportions, and would probably have been of low frequency tuning if designed in normal reinforced concrete. As such cantilevers off the block were notorious for vibrational troubles, it would be of interest to learn how Mr Keith had tackled the design of these using prestressed concrete.

Mr A. Atherton (Partner, Messrs C. S. Allott and Son) complimented the Author upon the thoroughness of his researches into the various mathematical treatments which could be applied in assessing the vibration characteristics in the design of turbo-generator foundations.

128. The Author stated in the early part of the Paper that previous experience with a partially prestressed turbo-generator foundation had proved the value of prestressing in avoiding shrinkage cracks. Presumably the argument here was that in the blocks referred to no shrinkage cracks had appeared, whereas presumably they had in the Author's experience on unprestressed foundation blocks. Mr Atherton suggested that this argument was not absolutely conclusive, as there were undoubtedly a large number of non-prestressed foundation blocks in existence which showed no signs of cracking.

129. He would perhaps go even further and say that the application of prestressing forces on a structure as complicated as a foundation for a large set could in itself set up secondary tensile stresses of such magnitude as to induce cracking. This was obviously appreciated by the Author in that he did give some indication of the measures adopted to relieve the secondary stresses which prestressing of the longitudinal and transverse beams had induced in the columns.

130. In the absence of any more detailed explanation of these measures, however, it was difficult to appreciate how the operation of jacking the bases of the columns inwards could be carried out with sufficient accuracy to ensure that the movement was just sufficient to obtain the desired effect. Mr Atherton asked whether the Author could state at what stage this jacking had been carried out and what allowances had been made for the shrinkage and creep which could have been expected to occur after the jacking operation.

131. With a view to establishing some data on the rate at which shrinkage in large concrete members could occur, Mr Atherton, in collaboration with Dr D. A. G. Reid, M.Sc., M.I.C.E. and Sir William Glanville, C.B., C.B.E., D.Sc.(Eng.), Ph.D., F.R.S., P.P.I.C.E., had initiated more than 25 years earlier the installation and reading of strain-gauges in a turbo-generator foundation with members having dimensional characteristics similar to those described in the Paper, and he had described the results of those observations in some detail in the discussion on Mr Rae's paper (this issue, pp. 375-390).

132. Shrinkage had been seen to continue over a period of about 8 years and had been only about half complete at the end of one year. These observations, coupled with the fact that creep strains would be induced by prestressing, implied that the measures referred to by the Author to eliminate the effects of shrinkage and creep once the machine was installed were not completely adequate if, as stated by the Author, differential movements of the order of 0.005 in. were critical.

FIG. 16: COUPLED OSCILLATIONS OF A T/A BLOCK.

 G_U = Mass of Turbo-Alternator and Top of Block. G_L = Mass of Block Footing.

$$C_U = \text{Block Stiffness} = \frac{4\pi^2 n_U^2 G_U}{g}$$

$$C_L = \text{Soil Stiffness} = \frac{4\pi^2 n_L^2 G_L}{g}$$

(a) On Rigid Support

$$(2\pi n_U)^2 = C_U / G_U$$

(b) On Elastic Support

$$(2\pi n')^2 = \frac{1}{2} \left(\frac{C_U}{G_U} + \frac{C_U + C_L}{G_L} \right) \pm \sqrt{\frac{1}{4} \left(\frac{C_U}{G_U} + \frac{C_U + C_L}{G_L} \right)^2 - \frac{C_U C_L}{G_U G_L}}$$

[When ground is very soft, $C_L \rightarrow 0$ and $(2\pi n')^2 \rightarrow C_U \left(\frac{G_U + G_L}{G_U G_L} \right)$ or zero

$$\text{and } n'/n_U = \sqrt{\frac{G_U + G_L}{G_L}}]$$

133. Mr Atherton did not believe that differential movements of this order were quite as critical as had been stated, and it was doubtful whether a concrete foundation block of this type and size could be designed to ensure that differential movements of this magnitude did not occur.

134. It was obviously important that differential movements must be limited as much as possible and, although a lot could be done by the application of sound basic principles by the structural designer, the large increase in size in generating sets in recent years seemed to be stretching the available theory and construction techniques to the limit. It was suggested that the time had been reached when a more fundamental approach to the problem was required. This could take the form of a complete re-consideration by the plant designers of the layout of the machines, with the object of reducing some of the present overall dimensions and introducing some discontinuities into the structure. It could possibly be taken even further, to the extent of considering afresh the possibilities of utilizing steel to a greater extent than had been recent practice in the construction of these foundations.

135. The main advantage which the Author claimed for prestressing was that by its use it was possible to adjust the natural frequencies of vibration of the members to avoid resonance, and that this could be achieved by increasing the total deflexion of the columns. In view of the fact that the additional deflexion due to prestress was not associated with any change in mass it was difficult to conceive how this could produce the changes in frequency which were claimed. The Author was asked whether he would comment on this point, as it was of major significance in relation to the Paper as a whole.

136. Mr Atherton thought it would be a very interesting supplement to the Paper if the Author could at this stage give some indication of the actual movements and deflexions which had taken place in the foundation, as compared with estimated values. Such information could be valuable in assessing the validity of the theoretical approach.

Mr I. P. Haigh (Research Engineer, Sir Alexander Gibb and Partners) commented that many of the points that he had contributed to the written discussion of Mr Rae's paper (this issue, pp. 375–390) were equally applicable to the Paper presented by Mr Keith; he had referred in that contribution, for example, to uncertainties in applying elaborate vibration analyses to turbo-alternator blocks, to the value of Young's modulus in concrete subjected to vibration stresses, to the frequencies of the forced oscillations in a block, and to the significance of shear and section rotation influences on the fundamental frequencies of transverse vibrations in stiff structural members.

138. Bearing in mind the relatively unimportant part played by the reinforcement in the vibration design of a reinforced concrete member, it seemed open to question whether or not the imposition of an axial stress by prestressing elements would make any appreciable difference to the natural frequencies of the substantial sections adopted in turbo-alternator blocks.

139. In §§ 11, 12, Mr Keith had described the German Code of Practice recommendations for determining the dynamic forces of a block as being the only fairly accurate guide to date. The equivalent static force was compounded of an assumed eccentricity of the rotor, the square of the machine speed, a factor of 10, a fatigue factor of 3 and a dynamic multiplier. The constant fatigue factor (Ermüdungsbeiwert in the original German) was a curious entry of obscure purpose; the dynamic multiplier was derived from lengthy calculation and included a 10% adjustment of the calculated natural frequency towards the machine speed to allow for inaccuracies in the calculations; finally, an upper limit of 15 was placed on the value of the equivalent static force of a 3000 rev/min machine to make allowance for damping. This upper limit corresponded, therefore, with a maximum acceleration during vibration of 15g. It was of interest to note that the foundations for some of the largest turbo-alternators now under construction in Britain had been designed on the direct assumption of accelerations of 5g on the turbine rotors and of 10g on the generator rotor.

140. In § 19, Mr Keith quoted a formula for calculating the fundamental natural frequency of a portal frame. This formula did not appear to be as satisfactory as that used by Mr Rae¹² (§ 76 of his paper) since it made the incorrect assumption that all points in the upper beam of the frame oscillated with the same amplitude as the mid-point, where the amplitude was normally greatest. In § 20, the conversion from centimetre to inch units appeared to be incorrect; the expression should have been divided by, and not multiplied by, the square root of 2.54.

141. The important issue of coupled oscillations was mentioned briefly in § 22. Some British civil engineers had quoted German sources, possibly inaccurately, in using this argument. If a vibrating structure had a natural frequency of n when supported on a rigid footing, and if it had the frequency of n' when supported on infinitely soft ground, it would then follow for any soil conditions likely to be experienced in practice, all having some intermediate stiffness, that the natural frequency would lie, conveniently, between n and n' . As would be seen from Fig. 16, the coupled frequencies did not lie between these limits of n and n' ; instead, two frequencies were introduced, one lying below n and the other substantially above n' . While Mr Keith did not use the foregoing argument himself and indeed actually mentioned the two coupled frequencies, it appeared to be incorrect that the lower frequency could be neglected and that the higher frequency was usually about 30% above n . If G_u happened to be equal to $0.69 \times G_L$, the upper frequency n' would start at $1.30 \times n$ and rise markedly above that value on stiffer ground. Similar families of curves could be drawn for different ratios of G_u to G_L ; at High Marnham, where the footing extended to 29 ft below basement level and was therefore unusually massive, G_u was apparently only equal to $0.32 \times G_L$, so that the minimum value of the upper frequency was $1.15 \times n$.

142. In German practice the coupling factor was related specifically to the coupling of the structure and the subsoil. Unfortunately coupling was also experienced between many pairs of structural elements in a block, between a slab and its supporting beam or between a beam and a column, for example. In any block, many such pairs were possible, and it was conceivable that local resonance could occur as the result of coupling a high-tuned member with a low-tuned member, either of which, considered singly, would otherwise have been satisfactory.

143. It had been this consideration, and others, that had led to the suggestion recently put to the Central Electricity Generating Board that the subject of vibrations in turbo-alternator blocks deserved fundamental study. It might well be that a new approach, using forced oscillation analysis, would be worthwhile; efforts would then be concentrated upon trying to derive amplitudes at assumed forced frequencies, instead of trying to determine natural frequencies and the incidence of resonance.

The Author, in reply, wrote that he recognized the validity of the main criticism made by Mr Rae and other contributors to the written discussion. This could be stated simply by saying that the static precompressive force could not be added to the dynamic loads and the vibrating mass of the block.

145. Mr Rae's brief references to ordinary reinforced concrete of the same cross-section, and to creep, raised two fundamental points. The first was the objectives aimed at in prestressing the High Marnham turbo-blocks. As indicated in § 2 of the Paper these were elimination of shrinkage cracks and of tensile stresses resulting from a state of near-resonance. This was outlined in §§ 26(1)–(5) and 59–61. This was, in point of fact, carried a stage further by prestressing to such an extent as to ensure a residual compressive stress of approximately 200 lb/sq. in. in all parts of the block under all conditions of loading that it was possible to predict. The subject of direct influencing of frequencies by prestressing was considered purely as a corollary by the Author and was not one of the original objectives of the prestressing.

146. The second fundamental point raised by Mr Rae was the question of additional creep due to prestressing. Mr Rae would no doubt agree that only differential movements were of some importance, since they affected the levels of the bearings

relative to one another and therewith the shaft alignment. Since the creep strain was related to the compressive stress, it was apparent that creep movements in the columns could be equalized by suitable adjustments in the prestressing forces. The fact that the creep was magnified by prestressing was, therefore, not necessarily of any great significance.

147. The Author agreed with the comments made by Professor Robertson and by Dr Allen. However, Professor Robertson's point regarding increased stiffness and an increase in the frequency, a point also made by Professor Horne, raised the question of interpretation. If shrinkage cracks opened up after construction of the block, the actual frequencies encountered would be nowhere near the calculated frequencies. Prestressing would then ensure that the actual behaviour of the block resembled rather more closely the predicted behaviour. It could, therefore, be somewhat misleading to state that the frequency would be increased.

148. The transformation of Rausch's formula into one with d expressed in inches was indeed wrong. A correction had been published. Regarding Professor Horne's comments on Fig. 8, the curves were meant to be only illustrative, the upper curves representing approximately upper harmonics.

149. In the calculations in the Appendix (§§ 66–70), the Author had considered, for convenience' sake, the simple case of a beam with a purely axial load, instead of the actual case of curved and straight cables in an encastre beam. The final conclusion reached by these calculations was similar. The deflexions produced in beams of a size such as the condenser beams would indeed be small enough to enable the cables to remain straight when freely stretched and left ungrouted. The Author agreed that a prestressing force which approached the Eulerian load did not lead on to lateral buckling if the cables were grouted up under elastic conditions.

150. The hypothetical case of the inverted portal stated by Mr Batson was certainly an interesting one! It was, however, all a question of the definition of deflexion. This had been used merely as another way of defining a load. The Author agreed with the importance of differentiating between dynamic loads and the static load produced by prestressing.

151. The Author had stressed that because of the irregular shapes, and by virtue of the nature of concrete, the calculations could only be approximate. He did not claim that a turbo-foundation could be designed using the most precise scientific methods, as Mr Wood and Mr Schlesinger appeared to imply. However, there was no doubt whatever that a rule of thumb could be improved upon. Reliance on large sections with irregular shrinkage cracks often occurring in awkward places, seemed questionable. Moreover, there would be an appreciable increase in differential settlement. This was a subject which had to be considered in conjunction with natural frequency and inherent damping.

152. The raft calculations in § 48 were based on the classical treatment given by Timoshenko. Its limitations had been stated most carefully. Calculation of the natural frequencies of different modes of vibration, as indicated in the Appendix, also followed the normally-accepted methods. The limitations of these idealized modes of vibrations were stressed repeatedly by the Author.

153. The full concrete section was effective by virtue of sufficient pre-stressing. The merits of low-tuned foundations were decreased risk of resonance as compared with a high-tuned foundation. Detuning had not been aimed at, as the Author had already explained in his reply to Mr Rae.

154. The thickness of the raft foundation was not determined solely by either vibration behaviour or the nature of the underlying strata, as Mr Crowder appeared to imply. It had to be considered in conjunction with the layout and size of the cooling water aqueducts and the turbine house column foundations. These factors showed a solid raft founded on the hard marl to be preferable. Coincidence as far

as possible of the centres of gravity of the raft, superstructure and machinery was certainly important, to avoid tilting of the foundation.

155. The question of tuning was bound up not only with the objections referred to, but also with the initial degree of prestress. If this was already high, as was the case with the High Marnham blocks, then further tensioning would produce little or no result.

156. It was probably fair to say that assessment of effective length and degree of end fixity of beams was intuitive rather than exact. The curves produced by Mr Crowder appeared to be very useful for making quick preliminary assessments.

157. Mr Elvery's account of an experimental investigation into the effect of a sustained prestressing force on the natural frequency of concrete columns was of great interest. It would be interesting to find out why, after initial recovery under load, the dynamic modulus should undergo a second reduction after removal of the prestressing force. Resonance tests had not been carried out on the High Marnham blocks. Fig. 8 showed approximate harmonics. Large frequency reductions were not claimed.

158. Mr Holloway had referred to the advantages offered by prestressing of the block. The Author would like to add that these were principally reduction of differential settlement and elimination of shrinkage cracks and tensile stresses caused by a state of near-resonance. Mr Holloway had drawn a clear distinction between cables which were grouted up and those which were not, and this had clarified the position.

159. Mr Waller's account of the use of laminated rubber bearings in the foundations of variable speed machines was extremely interesting. His lucid remarks on the choice of massive foundations helped to clear up a number of misconceptions. Generally, however, the choice of the size and type of foundation was not left to the civil engineers, but was determined largely by the plant requirements.

160. The new approach to turbine design outlined by Mr Aggett promised to simplify considerably the construction of the turbo-blocks, with a probable saving in cost and time. It also had the advantage of lowering the height of the turbine house operating floor, crane and roof.

161. The shape of the steam chest and throttle valve cantilevers was determined largely by plant requirements, as was the case with most parts of the block structure. They were indeed of low frequency tuning (as shown in § 51). Prestressing eliminated tensile stresses and produced a residual compressive stress in these cantilevers.

162. The Author appreciated Mr Atherton's observations. The jacking of the column bases had been done after completion of the horizontal stressing, and before stressing of the vertical cables (with the exception of a relatively small number of cables in one pair of inner columns which were anchored off at an intermediate level and had to be stressed at an earlier stage). The movement was intended to equal the elastic compression and the eventual total shrinkage and creep movements in the beams. Differences between top and intermediate level beams were taken into account. Initially, therefore, moments would be produced at the joints. These would disappear in time.

163. Mr Atherton's investigations into shrinkage and creep strains were of very great interest. A shrinkage period exceeding eight years surprised the Author, as it must have done many engineers, who hitherto had assumed that after three to four years roughly 90% of the shrinkage movements had taken place. It would appear, therefore, that a case could be made out for prestressing most larger types of blocks, with a view to avoiding shrinkage cracks. As the Author had already stated in his reply to Mr Rae, differences in creep strains could be minimized by suitable adjustment of the prestressing force. Shrinkage could not be controlled, but provided the columns were approximately of equal dimensions, shrinkage would not give rise to undue differential settlements.

164. Owing to relocation of settlement gauges the series of readings taken to date extended only over a comparatively short period. They gave the settlement of various points on the block relative to one control point on the block. The limited number of readings available showed only negligible amounts of relative settlements.

165. The Author agreed with Mr Haigh that the value of Young's modulus presented a great deal of uncertainty. The assumed acceleration of the largest turbo-alternators now under construction were of great interest. One ought to bear in mind that the provisions of the German Code had been in existence for quite a few years. As far as the Author had been able to ascertain, these provisions were based at least in part on experimental work carried out nearly 35 years ago. Mr Haigh's observations on coupled oscillations and the curves produced by him were of great interest. Lack of space had not permitted enlarging on this subject in the Paper. Undoubtedly coupling took place between many adjacent parts of the structure. No part could be treated in isolation.

166. Damping had a very much greater effect on amplitude than on natural frequency. For this reason it was to be hoped that a great deal more information on damping characteristics based on actual measurements would become available. Data on amplitudes would undoubtedly be of great interest to the General Electricity Generating Board.

REFERENCES

10. R. H. ELVERY and M. FURST. The effect of compressive stress on the dynamic modulus of concrete. *Mag. concr. Res.*, No. 27, Nov. 1957, pp. 145-150.
11. A. PADUART. Application de la théorie des vibrations au flambement et au déversement latéral des poutres précontraintes de grande portée. *Science et Technique*, No. 10, Brussels, 1944.
12. F. A. RAE. Design and construction of a reinforced concrete foundation block for a 200-MW generator. *Proc. Instn civ. Engrs*, vol. 22, 1962, pp. 41-56.