A design method for grillages

by

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Mr G. Sved (Reader in Civil Engineering, University of Adelaide) remarked that although Dr Percy's paper gave an excellent method of designing a grillage, it gave no assurance that the design would be an economical one, nor did it give any guidance regarding changes to be introduced to improve the design. Mr Sved put forward two suggestions to remedy this situation.

42. The effect of adjusting the variables under the designer's control could be ascertained by expressing the weight of the structure as a function of these variables.

43. In considering the first example in the Paper,

Let P be the force transmitted at E to member BEH; the force transmitted to DEF is then $R = 6000 - P$.

Let also the end moment and torque applied at B to BEH be T_1 and T_4 ; These would appear as torque and moment at B on ABC.

Similarly moment and torque on CFK would be T_2 and T_5 , while on ADG at A, T_3 and T_6 .

44. All moments, torques and shearing forces on the structure could be expressed in terms of $T_1, T_2 \dots T_6$ and P , if a symmetrical structure were assumed. Also $T_1 + T_2 + T_3 = 0$ from the torsional equilibrium of member ABC.

45. Once the bending moment and torque diagrams had been drawn (in terms of the unknowns), the possible critical sections could be selected; e.g. on member BEH the critical section would be B (bending moment T_1 , torque T_4) or at E (bending moment at $5P - T_1$, torque T_4).

46. Interaction curves (at least over a limited range) could be expressed for the i^{th} member in the form

$$\sigma_{\text{yield}} = \frac{K_1}{d_i^2} \sqrt{M_i^2 + a_i T_i^2} \quad \dots \quad (6)$$

where K_1 and a_i were factors depending on the shape of the section selected and on the failure criterion, d_i was a typical dimension of the section.

47. Since, for a given shape, the weight of the section was proportional to d_i^3 , solving equation (6) for d_i the weight of an arbitrary member could be written as

$$W_i = w_i d_i^3 = w_i \left(\frac{K_1}{\sigma_{\text{yield}}} \right)^{2/3} (M_i^2 + a_i T_i^2)^{1/3}$$
$$W_i = S_i (M_i^2 + a_i T_i^2)^{1/3} \quad \dots \quad (7)$$

where

$$S_i = w_i \left(\frac{K_1}{\sigma_{\text{yield}}} \right)^{2/3} \quad \dots \quad (8)$$

48. Since all M and T values were expressed in terms of the variables under the designer's control and all S values for the individual members had been calculated, partial differentiation provided the answers regarding the effect of changes.

49. To illustrate the method of calculating partial derivatives, a fictitious structure consisting of three members, j , m , n , was used; it was assumed that the torque T_j in member j affected the moment in member n so that $M_n = M_{no} - b_n T_j$, where b_n was a constant.

50. In this case

$$W = W_j + W_m + W_n = S_j(M_j^2 + a_j T_j^2)^{1/3} + S_m(M_m + a_m^2 T_m^2)^{1/3} + S_n[(M_{no} - b_n T_j)^2 + a_n T_n^2]^{1/3} \quad (9)$$

$$\begin{aligned} \frac{\partial W}{\partial T_j} &= \frac{2}{3} a_j T_j S_j (M_j^2 + a_j T_j^2)^{-2/3} - \frac{2}{3} b_n (M_{no} - b_n T_j) S_n [(M_{no} - b_n T_j)^2 + a_n T_n^2]^{-2/3} \\ &= T_j \left\{ \frac{2}{3} S_j (M_j^2 + a_j T_j^2)^{-2/3} + \frac{2}{3} b_n S_n [(M_{no} - b_n T_j)^2 + a_n T_n^2]^{-2/3} \right\} \\ &\quad - M_{no} \left\{ \frac{2}{3} b_n S_n [(M_{no} - b_n T_j)^2 + a_n T_n^2]^{-2/3} \right\} \quad (10) \end{aligned}$$

51. In most cases it was sufficient to know the *sign* of the multipliers of T_j and M_{no} ; the multiplier of T_j was always positive, while the sign of M_{no} depended on the sign of b_n . The direction in which W changed with T_j could be found thus in many cases without evaluating numerical values (i.e. in cases where both terms were positive or where by inspection one term could be seen to outweigh the other).

52. For a certain class of structures the statically determinate structure was the lightest one.⁴ Although all conditions required by the proof given in the paper cited were not satisfied by the grillages of Dr Percy, it was likely that for a single load a statically determinate grillage would be not far from the minimum weight.

53. Two statically determinate grillages were investigated:

(a) Omit BEH and its two supporting members, ABC and GHK. The resulting structure consisted of the following simply-supported members:

DEF	10 in. $\times$ 4½ in. $\times$ 25 lb/ft,	30 ft	750 lb
ADG	6 in. $\times$ 3 in. $\times$ 12 lb/ft,	20 ft	240 lb
CFK	8 in. $\times$ 4 in. $\times$ 18 lb/ft,	20 ft	360 lb
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			1350 lb

(b) Omit DEF and its supporting members:

BEH	9 in. $\times$ 4 in. $\times$ 20 lb/ft,	20 ft	400 lb
ABC	8 in. $\times$ 4 in. $\times$ 18 lb/ft,	60 ft	1080 lb
GHK			
			<hr/>
			1480 lb

These weights compared very favourably with Dr Percy's designs. (First design: 580 lb in rolled sections, 1586 lb in tubes, total 2166 lb; second design: 550 lb in rolled sections, 1594 lb in tubes, total 2144 lb.)

The Author, in reply, remarked that the Paper described a method of writing down an equilibrium system of internal forces for an indeterminate grillage. Mr Sved had pointed out that it gave no guidance to the designer in an important aspect of plastic design by the lower-bound theorem, namely the finding of an *optimum* equilibrium system. He was to be thanked for his interesting comments on this part of the design process.

55. Mr Sved suggested a determinate structure for material economy. When the design loading was appropriate, that was good advice. The economy of his (symmetrical) determinate designs over the Author's designs was striking, and the Author had thought that much of this economy might have been due to the fact that the symmetrical determinate structure was torsion-free, but this was not so. If, in example 1, the load at E was carried by the unsymmetrical determinate frame formed by the members CK, FD and AD only, supported by C, K, and A, the member FD carried torsion and bending. A design using a tube for this member (10 in. o.d. $\times$ ¾ in.), and RSJs for the others, weighed 1395 lb.

56. In the context of the Paper, however, it was somewhat irrelevant to discuss the economy of determinate structures since a designer would use the method only for a structure which was statically indeterminate. In such a case, Mr Sved suggested choosing certain forces as redundants and determining the sign of the change in the total weight of the structure consequent on a change in each redundant. This was indeed what the designer had to try to do, but the unwieldy equation (10), derived for a simple system with three members and one redundancy, was likely to discourage him from formalizing the process in a real case.

REFERENCE

4. G. SVED. Redundant weight of certain redundant structures. *Aust. J. appl. Sci.*, vol. 5, 1954, pp. 1-9.