A comparison between prestressed-concrete and reinforced-concrete beams under repeated loading

by

Stephen Charles Clavell Bate, Ph.D., B.Sc., M.I.C.E.

Mr A. J. Harris (Senior Partner, A. J. & J. D. Harris) said that although the Paper was exceedingly well documented, it was inevitable that there were certain details which were not given but on which information was of interest. Was anything known of the fatigue characteristics of the steel? Was anything known of the fatigue characteristics of concrete in compression? It would be attractive with these composite materials, where the interaction of the basic constituents was so complex, to start off with known characteristics of the two components.

48. Again, the Author referred to failure due to the yield of the steel, and Mr Harris found this very interesting. Did it imply that the fatigue failure of the steel was associated with a large yield? Normally this was productive of a brittle failure. He had in mind a series of fatigue tests done by Zercavins, who found fatigue failure caused by brittle failure of one or two wires in the bundle. Mr Harris would be grateful if the Author could enlarge on the nature of the failure. It was very interesting that after many millions of applications of loading on the prestressed-concrete beam, the apparent or the virtual modulus of the beam decreased, whereas that of the reinforced concrete beam increased. Would the Author care to comment on any possible explanation of this?

49. The last of Mr Harris's points of detail referred to Fig. 9 and § 33. The Paper was a very full one and required a good deal of digesting, but this was something he had been unable to digest. He did not know what was related on the two figures in Fig. 9 and he was completely unable to gather what had been calculated, what had been interpolated and what were the experimental data in this interesting paragraph.

50. Turning to a slightly more general consideration, it was very interesting to find that novel materials such as prestressed-concrete, coming as they had in the laboratory age, had provoked investigation into other materials. Reinforced concrete had been in existence for a very long time. Had any substantial work been done on the fatigue strength of reinforced concrete up to now? He had not come across any, but now they were confronted with the sudden discovery that reinforced concrete, against all theoretical supposition, was a material with a remarkably high fatigue strength. That was true also of prestressed concrete, but this was not unexpected.

51. They would have to ask themselves what they should do with this information. The Building Research Station had very properly been producing—a selection of the publications was given in the bibliography to the present Paper—some extremely interesting and significant information on the characteristics of reinforced concrete, prestressed concrete and other structural materials. What should practising engineers do with it? They had here a material the behaviour of which gave a fatigue limit of the order of 0.7 to 0.8, a remarkably consistent rupture strength, and

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elsewhere the Author and his colleagues had produced information indicating that there was a substantial uniformity of strength even with mass production continued over a period of many years.

52. Those were characteristics of a structural material which seemed to him to be far too good for anything for which reinforced concrete and prestressed concrete were likely to be used. They were concerned here with millions of repetitions of load. Loading as officially laid down was often such that they might well ask whether the design loading was ever going to come on the structure at all, let alone a million repetitions of that same load. He would suggest, therefore, as a sort of pendant consideration to the exceedingly interesting and instructive Paper which the Author had submitted, that they should ask themselves whether it did not call for a drastic revision of their codes of practice or of their loadings.

Dr P. W. Abeles (Consulting Engineer), while congratulating the Author on a very interesting Paper, pointed out that all his tests had been based on post-tensioned tendons which were positioned about two-thirds of the depth of the beam, and therefore the crack distribution was very unfavourable. Yet the essential point with regard to fatigue resistance and to ordinary resistance on static loading was the crack distribution. Looking at Figs 11 and 12 it would be seen that there was an excellent crack distribution in Fig. 12 (reinforced concrete beam), where there was only a very small cover, but there was a relatively bad crack distribution in Fig. 11 (beams with the post-tensioned tendons). On the other hand, tests on beams with pre-tensioned tendons which were well distributed showed an excellent crack distribution, perhaps better than in Fig. 12, and therefore an even better behaviour under fatigue of the prestressed concrete would be expected.

54. The Author's conclusions, which were very interesting, ought to be limited to post-tensioned tendons and not generalized, because Dr Abeles was convinced, based on experiments with pre-tensioned tendons, that the behaviour of prestressed concrete where the tendons were close to the bottom fibre or there was non-tensioned distributed steel close to the bottom fibre would be very much better. Similar tests had been carried out in the U.S.A. by the A.A.S.H.O. with loading by impact and fatigue, and the beams with pre-tensioned wires had behaved excellently under a tensile stress of 750 lb/sq. in., whereas the beams with post-tensioned tendons, having a larger cover, had shown similar crack distribution to that in Fig. 11; their behaviour had been quite different and wide cracks occurred after a loading of one million repetitions of impact. While, therefore, the Author's tests were very interesting and provided some material for the comparison of reinforced concrete and prestressed concrete, it was important to amplify them with tests with pre-tensioned tendons.

55. Some tests of that kind had been carried out by British Railways. In one case 7 million repetitions were applied between a concrete stress of 1500 lb/sq. in. compression and 250 lb/sq. in. tension before the cracks became visible, whereas in another case the cracks became visible for a range between concrete stresses of 250 and 750 lb/sq. in. after 700 000 repetitions of loadings. Dr Abeles thought that this range of stress was of great importance and gave much more information than the proportion of the working load. The greater the range of stresses in the concrete in pounds per square inch between compression and tension the less, obviously, was the capacity to resist high concrete tensile stresses in fatigue, and from that point of view it was important to refer both to the range and to the maximum concrete tensile stress. He agreed very much with the Author's statement that when the cracks were very narrow, say 0.005 in. or less, there was very little danger that great increase in crack width would occur due to fatigue loading, but if the crack width at first loading was over 0.010 in., there was great danger. That had been shown by some recent tests.

56. He was very glad to see from § 21 that the Author had considered the possibility that cracking was not dependent on the magnitude of the prestress but on the percentage of reinforcement, because originally it had been stated that the greater the

prestress the less was the stress at which cracks occurred, but in reality this related only to the visibility of cracking, and micro-cracks developed as soon as the tensile strength in the concrete was reached and the larger the percentage the later the cracks became visible. That had been pointed out by the speaker. The Author had made very important comparisons between prestressed and reinforced concrete, but in the view of Dr Abeles these should not be generalized but considered only as comparisons between post-tensioned tendons and reinforced concrete and not as general comparisons between prestressed and reinforced concrete.

Dr M. F. Kaplan (Director of Research, Civil Engineering Research Council) said that in § 7 the Author mentioned that special anchorage blocks had been used in the test specimens. Were these special blocks in any way extraordinary in relation to practice? Dr Kaplan felt that in practice the anchorage could be the weakest link in the case of prestressed concrete beams subject to repeated loading.

Mr R. W. Pearson (Ministry of Public Building and Works), while agreeing that the Paper contained much valuable information, felt that the conclusions were perhaps slightly misleading and might give false comfort to the practising engineer who of necessity had not the time to study closely all the Papers which were published, and tended to accept their conclusions as sound advice.

59. At the end of § 34 there was the statement that 'the ratio of the failing load for one million repetitions to the static failing load of 0.7 to 0.8 would be unlikely to vary appreciably for beams made with the same materials.' The last four words should be in large type and should be repeated in the conclusions in respect of all these tests. To his mind, that remark was more important with regard to reinforced-concrete beams than with regard to prestressed beams.

60. The important thing to aim at was to find the worst cases likely to occur using material which complied with the appropriate British Standard. Taking them in the order in which they were dealt with in the Paper, and considering first prestressed concrete, he had not much quarrel with what was said and did not think that there was much wrong with the advice or the methods, but he would point out that the ultimate failing load of the wire used was, of course, above the minimum, and this was the point that he had in mind: what would be the worst result which could be obtained with material passing the appropriate British Standard?

61. Coming next to reinforced concrete with mild steel reinforcement, the first thing that he had looked at was the Table, to find the properties of the steel used, and he noted that the yield stress was given as 43 500 lb/sq. in., or about 19.4 tons/sq. in., and this was, of course, appreciably higher than might in practice be obtained. He imagined that bars with a yield as low as 15 tons/sq. in. might be found and might perhaps have given a worse result. It would be nice to know and it might give another angle on behaviour. In accordance with C.P. 114 it would be permissible to stress the bars used to 21 750 lb/sq. in. and not the 20 000 used in the design. This could be treated as a guaranteed yield steel which was above the minimum, as it were. That would definitely have worsened the load factor ratio given.

62. Finally, taking the reinforced concrete with the cold-worked steel, from the tests described it was clear that the fatigue failure was dependent on the stress history of a bar, yet no information was given about the properties of the mild steel from which these particular bars had been made, and only in Appendix 2 was any mention made of the elongation, from which something might be deduced about the properties of the steel. It would be nice to start off with a low yield stress mild steel bar, cold work it to a proof stress of perhaps 80 000 lb/sq. in., and repeat these tests. This should result in a bar of the lowest possible elongation and presumably with the lowest possible fatigue strength. In his opinion, only by that sort of approach would it be possible to obtain results which would enable firm conclusions to be drawn of real use to the designer, and it might also lead to further changes in British Standards.

Mr A. Goldstein (Partner, R. Travers Morgan & Partners) observed that he had found the Paper extremely informative. He invited the Author to comment on some views relating to the effect of his work on the work of the designer.

64. The first question was why the figure of 1 m. repetitions had been chosen as a yardstick. Mr Goldstein believed that in continental Europe, again for no obvious reason, the figure of 2 m. was more common. That was, of course, purely a yardstick, but to use the same yardstick would be helpful for comparison. One million repetitions might sound a lot, and, taking Mr Harris's point about buildings, it would generally be agreed that fatigue was not a problem on normal buildings; that by the time it was likely to become a problem the building ought to have been pulled down in any case. In the case, however, of the design of small structural units of bridges etc., i.e. units that received design loading reasonably often, what did a million mean? A traffic lane of 10 ft width might take 10 000 vehicles on an average day. He should emphasize that he was considering only small structural units such as would approach design stresses. If there were 10 000 vehicles a day, there might be a maximum of 1000 a day which came near to the design load, and taking 200 'days' in the year, to be conservative, it would mean 200 000 occasions a year when something approaching the design load occurred on these small units. On larger units, of course, this figure would be too high.

65. That kind of approach showed that one million was not all that big in terms of structures which had to remain in position for a very long time.

66. He wished to emphasize the point made previously that one should know the stress range that the wire had to undergo in these tests. On an *ad hoc* basis and for one particular purpose he had been involved in some push/pull fatigue tests on wire. The work had been published in a paper by Brereton and himself.⁹ Prestressing wire in a structure usually did not have much stress variation, but their investigations on control specimens of good wire for quite large stress variations had gone up to 10 m. repetitions without failure and had still been going strong at termination. In considering the cases where the wire had broken under fatigue loading in the tests described in the Paper, he felt, as Mr Harris did, that it would be desirable to start with the wire and correlate the results on the wire with the results on the beam.

67. On a more philosophical point, it seemed from the tests that, in the case of long-life structures, fatigue reduced by a definite amount the maximum load-carrying capacity, though of course it was a very small proportion. Some of the load factors that the Author had demonstrated seemed to be low. That might just have been incidental to the test; but the philosophical problem facing the designer was whether to produce a structure on the 'opening day' on the basis of a notional load factor of 2 or 2½ built in, by dealing with all the known factors (fatigue, stress concentration and so on), and then add a load factor of whatever it might be to deal with the unknown things, or whether there were certain elements, such as stress concentration and fatigue effects, which could be dealt with by assuming that they would be taken care of by the load factor. The present Paper was perhaps not the right one on which to invite the Author to comment on this type of problem, but as more knowledge of the type presented was acquired, a decision on these issues was necessary.

Mr C. D. Crosthwaite (Freeman, Fox & Partners) supported what Mr Goldstein and Mr Harris had said and raised a few other points, one of which related to what Dr Kaplan had said about the special ends of the beams. Had those ends failed under the repeated load test and because of that test, or did they fail on the static test? If the failure was associated with the repeated load test, it looked as though more information was required.

69. Secondly, the tests with the high alumina cement concrete had presumably been done at 28 days. Were any of those beams still in existence? Was the Author in a position to carry out tests at a greater age? In Mr Crosthwaite's view, high alumina cement concrete tended to become brittle with age.

70. Finally, the Author had begun his introduction of the Paper by saying that failures had occurred on railway sleepers and diving boards. It would be interesting to know what sort of failure had occurred on a diving board and exactly what had happened!

Mr G. R. Lewis (British Railways Research Department, Derby) said he took the greatest interest in the results from fatigue tests. At Derby they had been conducting tests, mainly with pre-tensioned beams, with a much better stress distribution, and he supported what Dr Abeles had said on that point. They had found that there was a considerable difference between the stresses that produced what they called visible cracks (of 0.001 in. in width and over) and the stresses which would produce failure. It seemed to him that before it was possible to go any further and estimate the fatigue endurance or resistance, in particular, of pre-tensioned beams, it was necessary to know what could be considered as acceptable cracking. It must be borne in mind that these cracks would always close when the load was removed.

72. To illustrate the degree of improvement that there could be if cracks were to be not more than 0.0001 in. wide, with what might be called an infinite number of cycles, 10 m. cycles, the nominal tensile stress in the concrete should not exceed something of the order of 650 lb/sq. in. If wider cracks, but not failure, were permitted, it would be permissible to go to something of the order of 1500–1600 lb/sq. in. nominal tension. There was a considerable difference there, and if cracking of that type could be permitted, considerable savings could be made on beams.

Mr F. Walley (Superintending Engineer, Ministry of Public Building and Works) referred to the Author's statement that cracks increased more rapidly in prestressed concrete than in reinforced concrete and wondered whether the Author would say that this was as true for pre-tensioned beams as for post-tensioned. The Author had tested pre-tensioned beams under fatigue conditions and would he be able to say whether or not he would draw the same conclusions?

74. Perhaps he would also say, because of the crack distribution shown in his illustrations, whether or not there was good bond for the wires. It seemed possible that since they were so widely spaced that the bond was not perfect. He might be able to say whether the crack distribution was of the kind that he obtained with post-tensioned beams generally. If so, it was a fair comparison between post-tensioned and reinforced concrete.

Dr F. G. Thomas (Deputy Chief Scientific Officer, Building Research Station) said that he had been interested in one or two general remarks which had been made and thought it might be useful if he were to say something about the purpose of the present and some similar papers coming from the Building Research Station and other research establishments from time to time.

76. When a particular research project had been completed, it was necessary, in general, (a) to see that the information obtained in the research was put on record, and (b) to consider what practical recommendations might reasonably be put forward as a result of the research findings.

77. In the present case the intention had been merely to put on record the results of a research project, and the Paper did not contain recommendations for designers. Another type of paper from the B.R.S. would perhaps contain fewer research data and more interpretation and recommendations for practising designers, but the stage at which recommendations could be made on the subject considered by Dr Bate in his Paper had not yet been reached.

78. It was likely that, later in the year, committees would be set up to revise the codes of practice for prestressed concrete and reinforced concrete, and he believed that these next revisions would be much more comprehensive and perhaps more philosophical than had been attempted hitherto. The basic principles of structural

design certainly required re-thinking. The codes had developed over a period of years and very few of the clauses in them had a logical scientific basis. Whenever a revision of codes was undertaken, further questions arose which could be answered only by research, and many of the items of research at the Building Research Station were devoted to getting information which, when properly interpreted in association with practising designers, could be incorporated in codes of practice.

Dr J. E. Spindel (British Railways) remarked that he had been led to speak by the impression which the figure of 1 m. seemed to have made on some people. He was concerned with the design of bridges. An investigation into the loading of small span members had shown that they were subject to something like 220 m. repetitions. What was of interest was that these were not repetitions of the maximum design load but in fact were only some 60% of it, and that led to the question which he wished to put to the Author. In a steel member, if the load was reduced, one knew what that did for the stress cycle, but in a prestressed concrete member which might be cracked under the maximum load and was at zero concrete stress at something less than the maximum load the stress cycle was not so easily determined. Could the Author say how this varied with the change from one condition to the other on a concrete beam and relate that to any data available on the inherent fatigue strength of the reinforcement?

The following contributions were received in writing:

Mr M. Sas-Skowronski (Civil Engineer, East African Railways & Harbours, Nakuru, Kenya) wrote that the Paper presented by Dr S. C. C. Bate revealed the extensive work accomplished by the Author in selecting, analysing and verifying experimental results. His aim was well orientated towards a goal of indisputable value to all structural engineers. Keeping the main purpose in mind, the average results obtained were satisfactory. The ratio of the maximum dynamically applied load P_d to the computed static ultimate strength P_u did provide a true indication of the relative intensity of each loading. The comparison on the basis of the relative strength between reinforced and prestressed concrete beams was very useful for the structural engineer, as he must face this problem very often.

81. However, very little was said on behaviour of bond under the repeated loading, and sheer problem was completely omitted.

82. It would be desirable and useful to present the results on the 'stress or stress ratio—number of load cycles—probability of failure' graph or on the modified Goodman's graph which would enable the designers to interpolate and extrapolate the above values for their own use. There should also be added the age of specimen when tested (this was believed to be at 28 days), curing conditions, type of aggregate, quality of the ingredients, proportions of cement, aggregate and mixing water.

83. It would be very interesting to know how modulus of elasticity of concrete was affected. Such information should be expressed in terms of an empirical constant. Such laboratories tested with good results should produce the equations over a wide range of variables, as in the design of reinforced and prestressed concrete, a knowledge of the elastic strain properties of the material was very important.

84. The writer had carried out extensive research on fatigue of the short reinforced and prestressed concrete beams and railway sleepers subjected to heavy loading and found that the sheer fatigue limit was about 40% of the ultimate sheer strength for the static load, and the sheer strength of concrete was the critical factor in bond failure. Splitting and cracking were important factors influencing the pull-out resistance of bonded bars.

85. If these factors were effective, no increase in load capacity of bond bars under dynamic loading over static loading should be expected. It was also noted that the

frequency between 70 to 380 cycles per minute had no practical significance. When the reinforcement was increased by 33%, the static strength was increased by 30%, but fatigue limit was increased three times (not magnitude of loading but numbers of cycles).

86. It was also found that the bond between mortar and aggregate decreased under the repeating loading.

87. Finally, the writer wished to know whether the changes and the concentration of stresses around the cracks were examined by X-rays (R. H. Evans' method) and, if so, if any results and photographs would be available in the *Proceedings*.

Mr R. Taylor (Lecturer in Engineering, Simon Engineering Laboratories, University of Manchester) wrote that Dr Bate had clearly stated that the tests were essentially for purposes of comparison. The writer would like to draw attention to this, since there was a danger that the conclusions drawn from the tests might be applied too widely. In the tests on the reinforced concrete beams, for instance, the stirrups were deliberately proportioned to preclude the possibility of a shear failure. In tests to compare the behaviour of the longitudinal reinforcement, this was, of course, desirable, but it must also be recognized that the behaviour of the longitudinal reinforcement could be affected by the inclined cracking in a beam. By over-reinforcing for shear, the inclined cracking in a beam was reduced.

89. The effect of inclined cracking on the behaviour of the longitudinal steel was shown in a series of repeated loading tests made by the writer at the Building Research Station.¹⁰ The aim of the tests had been to determine whether repeated loading caused shear failure in beams in which the stirrups were proportioned in accordance with some new proposals.¹¹ The beams so designed had rather less shear reinforcement than would have been required by the method of design in C.P.114. In none of these tests did shear failure occur, but the inclined cracking occasioned during the tests did affect the behaviour of the longitudinal reinforcement.

90. In this programme, four beams reinforced with deformed cold-worked mild-steel bars were tested. All failed by the brittle fracture of the longitudinal reinforcement, but in three of these beams fracture of the bars occurred not at a section of maximum bending moment, but within a shear span. Undoubtedly, the inclined cracking had affected the behaviour of these beams. Although two of these three beams contained longitudinal bars which were stopped-off where no longer required to resist flexural stresses, and this arrangement of bars undoubtedly affected failure of the continuous bars, failure of bars within a shear span occurred in the third beam in which all bars were continuous. It was probable that the lateral deformation of the longitudinal bars following inclined cracking (dowel action) caused secondary stresses to develop and promoted the brittle fracture of the cold-worked bars. The important point, though, was that this beam failed after nearly 2 million applications of a load range in which the maximum was only 65% of the static failing load; or, if the working load was taken as 50% of the static failing load, the beam failed when the maximum load was only 1.3 times the working load.

91. In the tests referred to, the minimum of the range was set at approximately one-third of the working load; it was noted that the Author adopted a value of half the working load. Since the fatigue resistance of concrete members was dependent on the range of loading, the load factors against failure quoted by the Author must be related to the minimum value of the range. This was clearly indicated by the Author in the test with repeated reversals of loading. But whereas such stringent loading conditions were, as suggested, unlikely, it was possible that many concrete structures, particularly prestressed concrete structures, would be subjected to ranges of loading in which the minimum of the range (the dead load) was less than half of the working load. Whilst it was very necessary in a research programme to be realistic, it seemed desirable to adopt the worst conditions that could occur, particularly as designers were apt to remember only the load factors quoted. From this viewpoint,

a value for the minimum of the range of loading of a quarter to one-third of the working load would perhaps have been preferable.

92. Would the Author justify the use of his formulae for determining the static strength of the beams? The implication of the relationship for the reinforced concrete beams, for instance, was that the average stress in the compression zone at collapse was equal to the cube strength of the concrete. While the writer would agree that the 'extra' two-thirds factor recommended by B.S., C.P.114 should not be adopted in determining the strength of a beam made in a laboratory, it seemed unlikely that such a high average stress as that implied by the relationship could occur.

93. Would it be possible for the Author to comment on the philosophy of load factors in concrete structures, with particular relevance to the effect of repeated loading? It could greatly facilitate the experimental work of research workers if some authoritative statement on this could be made.

The Author, in reply, wrote that the Paper was the third of three papers dealing with an experimental study of prestressed concrete under repeated loading. The research had originated from earlier work done in connexion with the development of prestressed concrete sleepers. That had suggested that further information was necessary, particularly on the behaviour of different types of cold-drawn prestressing wire. The work was later extended to include strand and was described in the first two papers.^{4,5} Since those investigations had been made on beams with pre-tensioned wires, the project was extended to beams with post-tensioned cables and included tests on reinforced concrete beams for comparison. As Dr Thomas had indicated, each of the Papers was concerned rather with the presentation of factual data on specific forms of test specimen, than with the more general interpretation of the information for practical use in design.

95. Mr Harris, Mr Goldstein, and Dr Spindel were of the view that the results of the research should be expressed in a form that could be directly applied by the practising engineer, and that they should be correlated with the fatigue strength of the materials taking into account the cycle of stress sustained by the steel. In the first⁴ of the three Papers already mentioned, the fatigue strength of cold-drawn prestressing wire had been given for tests made with the wire in an environment of concrete. The evidence suggested that for wire of the make and type used in the tests described in the Paper, one million repetitions of stress could just be sustained when the stress fluctuated between a minimum of $0.54 f_u$ and a maximum of $0.80 f_u$, i.e. a range of $0.26 f_u$ (where f_u is the tensile strength). In the present series of tests, the stress in the steel under the minimum load was about $0.60 f_u$ so that the maximum stress that could just be sustained for one million repetitions would be expected to be slightly less than $0.26 f_u$ greater than the minimum value, say $0.85 f_u$. The maximum stress for concrete under fluctuating compressive stress would be expected to have a value of about $2/3$ of the static strength.^{1,3,12}

96. If those values for maximum stress were substituted in the expression at the foot of Table 2, as providing an approximate but over-simplified means of calculating ultimate strength, it could be shown that the ratio of calculated strength in fatigue to the calculated strength under static loading ranges from 0.81 for beams with the smallest number of wires to 0.66 for beams with the largest number of wires, assuming that the materials have strengths of the average values of those actually used. That range compares with the experimental values of about 0.77 ± 0.05 which were apparently independent of the amount of steel in the section thus suggesting that the reduction factor applied to the strength of the concrete might have a higher value than $2/3$. Such an approach requires further substantiation, but it was clearly desirable that sufficient information should ultimately be available to be able to define fatigue failure in relation to the detailed requirements for numbers of cycles of loading and variables in the properties of materials as set out by Mr Sas-Skowronski.

97. Dwelling upon the modes of failure queried by Mr Harris, the Author

explained that prestressed beams stressed with four wires failed by fatigue fracture of the steel as was expected from the results of the earlier investigation, while those stressed with cables of 6 or more wires failed under repeated loading by crushing of the concrete, the disruption of the concrete having the same appearance as in comparable static tests as illustrated in Fig. 11. Examination of fatigue data for plain mild-steel suggested that the conditions of stress in the reinforcement of the reinforced concrete beams would be such that yield of the steel would occur before fatigue fracture could develop. In those reinforced concrete beams, failure of the concrete developed following yield of the steel and confirmed the earlier results of Lea.¹³

98. Had reversals of stress been introduced by testing plain mild-steel bars in the manner adopted for the cold-worked deformed mild-steel bars as described in Appendix 2, it was possible that a brittle fatigue fracture of the steel would have occurred. For the cold-worked deformed bars, a review of information on fatigue characteristics showed that fatigue fracture would occur before excessive elongation could lead to failure of the concrete; it further showed that reversals of stress within the permissible range given in C.P. 114 might cause failure as was demonstrated by the result given in Appendix 2.

99. At this stage, it was not clear how safety against fatigue failure could best be defined. One approach which lead directly from the experimental work was to adopt some value for a load factor against fatigue failure after a specified number of cycles of loading, as was discussed by Mr Taylor and by Mr Goldstein. These tests on prestressed concrete showed that the actual load factor obtained in fatigue was approximately $\frac{3}{4}$ of that for static loading. With a load factor of 2 for static loading, the load factor for fatigue loading was approximately $1\frac{1}{2}$. Since that value could be met by current design methods, it might be questioned whether that value would make a reasonable requirement, i.e. was it a useful requirement for structures subjected to repeated loading to be required to sustain one million repetitions of an overload of $\frac{1}{2}$ the total design load? A better definition of safety might be obtained by requiring a structure of that kind to sustain one million repetitions of a less severe over-load without reduction in static strength, and without visible cracking under normal working loads.

100. The Author thought that since prestressed concrete was designed both to comply with limitations of stress under working conditions and to provide a specific load factor against failure under static loading, one or other of those requirements might actually govern design. If the load factor requirements were adjusted to match the limitations imposed by permissible stresses then various relationships between load factor, permissible stress, and amount of prestressing steel in the section could be obtained; examples of such relationships were shown in Fig. 9, where permissible bending tensile stress had been taken as the criterion for calculating the working load.

101. The fuller explanation required by Mr Harris, of the Author, was therefore that the diagrams had shown the relationships between the prestress in the concrete, which is a function of the amount of prestressing steel, and the load factor obtained when the experimental failing load was divided by the calculated working load. The only limitation imposed in the calculations was that of permissible bending tensile stress, the selected values for which were shown against the curves. In the right-hand diagram the load factors were obtained from the static failing load for each beam at each level of prestress in the concrete, and in the left-hand figure the load factors were obtained from the loads estimated to cause failure after one million repetitions of loading for the corresponding beams.

102. Thus for the beams stressed with four wires giving a prestress in the concrete of about 1000 lb/sq. in. for no bending tensile stress, the load factor for failure under static loading was about 2.5 and that for failure under repeated loading was about 2.0. If the bending tensile stress were allowed to have a value of 550 lb/sq. in., then

the working load would have a higher value, and the load factors would become about 1.7 and about 1.3 respectively. For that particular beam a requirement for a load factor of 2.0 on static loading would limit the bending tensile stress to about 300 lb/sq. in. and the load factor against fatigue failure would then be about 1.6.

103. All the results for strength under repeated loading given in the Paper were, as Mr Taylor pointed out, dependent upon the value chosen for the minimum load of the range of repeated loading. The precise value chosen for the minimum load was probably of little significance for prestressed beams when fatigue failure occurred in the steel, provided that any cracks in the concrete that had occurred were closed under the minimum load, since its influence on the stress in the steel was relatively small. Its significance, if failure occurred as a result of crushing of the concrete in the prestressed beams, was likely to be more marked.

104. In reinforced concrete beams with plain mild-steel reinforcement, the fatigue limit for the steel would be influenced by the minimum stress. If as was usually likely for bending failure of the beams, the yield stress was below that limit, then failure would occur by yield of the steel. The fatigue limit for the beams was then unlikely to be much influenced by the value of the minimum load chosen for the repeated loading. Where beams were reinforced with cold-worked steels which fail by fracture in fatigue, the minimum load would be expected to have a substantial influence on the fatigue limit for the beams.

105. The Author referred to Mr Pearson's opinion that since the results had been obtained in tests on specimens made in the laboratory with materials of good quality, complying with the requirements of British Standards where appropriate, there was a possibility that the conclusions might be slightly misleading to the practising engineer. Since specifications for steel for prestressed or reinforced concrete did not have requirements for strength in fatigue, there might be a possibility that steels complying with those specifications could give less satisfactory results than those obtained in the investigation. The behaviour of the range of prestressing steels of British manufacture reported on in the earlier papers did not substantiate that suggestion. Review of information on reinforcing steel also did not indicate that substantially less satisfactory fatigue resistance was likely to be obtained. If the yield stress for the mild steel reinforcement had been 15.0 tons/sq. in. instead of 19.4 tons/sq. in., as in Mr Pearson's hypothesis the load factor against fatigue failure would have probably been reduced from 2.2 to 1.7; at the same time however the load factor against failure under static loading would have been reduced from 2.5 to 1.9 which might have been a less acceptable reduction.

106. For cold-worked steels, compliance with a requirement for a specific proof stress was usual, but control over the extent of cold-working, i.e. the strain history, which might influence fatigue strength, was not specific. It was therefore desirable that a wider range of types of higher tensile reinforcing steels should be tested. If the strength of concrete were less than was intended, it would seem likely that the reduction in fatigue strength would not have been greater proportionately than the reduction in strength under static loading, and the problem did not therefore differ from that for all concrete construction.

107. Dr Spindel and Mr Goldstein commented on the arbitrary number of repetitions chosen for setting the fatigue limit, and the Author, in his reply, stated that for those investigations, the figure of one million was selected as being convenient in testing. The fatigue machine was operated at 250 c/min and consequently one million repetitions were completed in about three days. To determine whether or not failure was remote if a beam survived one million repetitions, the test was continued for a further two million repetitions, which took about 9 days in all. If a larger number of repetitions had been set as the limit for fatigue failure, the overall time of the investigation would have been correspondingly increased since it was found impracticable to test at a more rapid rate.

108. Dr Abeles was concerned that the conclusions drawn from that Paper with

respect to cracking, should not be taken as applying to prestressed concrete members with pretensioned steel. In reply to Dr Abeles and Mr Walley who also discussed cracking, the Author wrote that the tests on beams with pretensioned steel, which were made in the earlier investigations, were not directly comparable with those described in that Paper. The results however suggested that beams with pretensioned steel showed better crack distribution with less rapid opening of cracks under load than beams with post-tensioned cables. When compared with the reinforced beams the comparison was less favourable to the beams stressed with pretensioned steel.

109. He then referred to Mr Walley's further comment on bond and cracking. The examination of beams showed that the grouting of the cables was satisfactory, and the poorer distribution of cracks as compared with beams with pretensioned steel was probably due to the poorer bond normally associated with post-tensioning, and to the greater cover to the prestressing steel concentrated in the cable.

110. The failure of the end-blocks of the prestressed beams, to which Dr Kaplan and Mr Crosthwaite referred, occurred during the stressing operations. For convenience in manufacture, therefore, special end-blocks were precast using the concrete of the same mix as in the beams and allowed to mature for a long period before use in the specimens. The failures were not associated in any way with the loading tests and there was no sign of failure in the end-blocks during the tests.

111. The tests on the beams made with high alumina cement were referred to by Mr Crosthwaite. At the time of test, those beams were between two and three months old, and at that age there was no indication of any deterioration in behaviour. None of those beams were in existence, but the main purpose of testing beams made with that cement had been served, and showed that conversion did not result from large numbers of repetitions of loading. Mr Crosthwaite also enquired about the reported failure of a diving board; there was however no detailed information on the occurrence.

112. Mr Taylor asked for justification of the formulae used for adjusting the strength ratios in Tables 2 and 3, for variations in the strengths of the steel and the concrete of the different beams in each series. It was not suggested that those formulae should be used in design; for the beams and methods of testing used in the study, they gave a closer approximation to the actual failing loads than the formulae accepted for general application in design, and were therefore accepted on that basis only.

113. The Author thanked Dr Abeles, Mr Lewis, Mr Sas-Skowronski, and Mr Taylor for the additional information given from their investigations on the behaviour of prestressed and reinforced concrete under repeated loading.

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