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The linear analysis of thin shallow shells†

by

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Discussion

Mr D. Dennington (Manager, George Wimpey & Co., Ltd) wrote that Mr Munro had based his calculations on the use of an elastic homogeneous material of uniform thickness and referred to it as concrete. Reinforced concrete was the accepted material for shell construction, for which these equations were intended for design purposes, and it was doubtful if the foregoing assumptions could be reproduced in practice.

50. A solution providing bending moments would lead to an economic design with varying steel areas, these would give varying flexural and area properties which in turn should affect the design more than they usually did for reinforced concrete work. Disregarding this fact, continuous equations (20) to (25) might not be considered for direct forces in a cracked reinforced section which was a condition likely to occur in shallow shells under asymmetrical loading.

51. In view of the above, would the Author not agree that load factor design based on ultimate theories and experimental work was more applicable to these designs?

52. As an extension of the Paper it would be interesting to compare it with revised calculations based on two steel membranes in the plane of the shell of equal thickness and a constant distance apart; these could be reproduced more nearly in practice using a constant-weight fabric reinforcement in the top and bottom of a concrete slab, with the concrete taking shear forces and supplying the main protective membrane. Apart from simplicity of construction details a closer relationship between theory and practice should be reached with such a design.

Mr R. S. Jenkins (Partner, Ove Arup & Partners) wrote that three of the early pages of the Paper were taken up with statements in primary and secondary terms of equations of equilibrium, stress-resultants, stress-couples and strains. Shallow shell theory depended on the consistent omission of all secondaries.

54. In the absence of a justification on an energy basis showing that secondaries were negligible in thin shells, Mr Jenkins was of the opinion that, today, a more acceptable thesis was to begin by saying that the shell was regarded as a lamina in a state of plane stress, with the inclusion of strains due to the normal displacement; combined with a plate in a state of flexure; the two being linked by the equation of equilibrium in the direction of the normal. Then, after the explanation regarding second order surfaces, the theory could have begun at equation (47).

55. As the subscripted notation was here not used for group methods, the Paper would have been easier to read had Love's notation for displacements (u, v, w) been adopted instead of ($u_\alpha, u_\beta, u_\gamma$).

56. These rather trivial criticisms, made in the interests of the general reader, did not detract from Mr Jenkins's opinion that it was a valuable Paper. It was gratifying to see a publication in Britain on a subject that was receiving much attention in other parts of the world.

† Proc. Instn civ. Engrs, vol. 19 (July 1961), pp. 291-306.

The Author, in reply, thanked Mr Dennington and Mr Jenkins for their interesting contributions. As Mr Dennington rightly stated, the analysis was for an elastic homogeneous material of uniform thickness. In addition, the analysis was for small displacements and Poisson's ratio was neglected. The latter could readily have been included, but since its square was generally negligible when compared with unity, its influence was small. In any case, its value for reinforced concrete was doubtful and was apparently a function of the stress tensor.

58. The analysis was strictly valid only for uncracked shells. However, some tests had been performed on a hyperbolic paraboloid shell model of reinforced mortar and the results compared favourably with the corresponding solution of equation (67) of the Paper. The shell was supported on four parabolic generators by deep gables and the tests have been described elsewhere.¹²

59. Mr Dennington's suggestion that the concrete shell should be considered as a "sandwich" construction would appear to complicate the analysis rather than simplify it.

60. The Author agreed that some form of limit analysis was required, but saw no conflict between this and the linear analysis. More information was also required about the non-linear elasticity and stability of thin shells, but these subjects, like limit analysis, were relatively in their infancy. The few available test results should serve as useful checks on any proposed limit analysis, but it would be unwise to derive an empirical ultimate load formula from these results.

61. It should be noted that the "membrane" (or extensional) theory did not lead, in general, to an acceptable complete solution for shells of negative Gaussian curvature, since the relevant second-order partial differential equation was of the hyperbolic type and could not be solved for a closed boundary.

62. In reply to Mr Jenkins, if a condensed notation were not used, economy of symbols was even more important and, in addition, it was desirable to use a single symbol for similar functions. Thus, all stresses were denoted by σ , strains by ϵ , and extensional actions by n , to give only three examples. The suffix notation served to distinguish the various members of each group. In using this simple convention, the Author, like most modern authors, had abandoned Love's notation. Therefore, there seemed little point in illogically retaining Love's convention for displacements.

63. Whilst agreeing that the earlier part of the Paper could be summarized briefly as stated by Mr Jenkins, the Author felt that the condensation would have presented greater difficulty to the general reader than any departure from Love's notation.

64. In conclusion, there was one correction to be made to the Paper. The right-hand side of equation (90) should be " $-Et\nabla^2 G_y$ ". The minus sign had been omitted in reference (11) and the error had been repeated in the quotation from this reference.

¹² J. Munro and B. M. Ahuja, "An investigation of the strain distribution in reinforced concrete shallow thin shells of negative Gaussian curvature". Symposium on Shell Research. International Association of Shell Structures. Delft. 1961.