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Stresses in reinforced concrete shields for nuclear reactors†

by

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Professor A. D. Ross and Mr G. L. England (Professor of Civil Engineering and Assistant Lecturer, respectively, at King's College, London) commented that the Paper was welcome because the literature dealing with thermal stresses in reinforced concrete was scanty. The problem was not, of course, confined to nuclear shields, for many reinforced concrete structures were subjected to thermal gradients, more or less severe, and the Paper was an interesting and useful study of the mechanics of the idealized elastic case.

46. However, they were interested in the actual stresses created by a sustained thermal gradient, and had tested a number of model beams in which curvature or warping was prevented but longitudinal deformation was not restrained. The specimens had been adequately instrumented so that there was a fairly complete picture of the temperature distribution and strains both in the concrete and steel.

47. The results of these tests were so startlingly at variance with the theoretical elastic solution that a typical example was presented in Fig. 8. The case was that of a

FIG. 8

† Proc. Instn civ. Engrs, vol. 16, p. 259 (July 1960).

singly reinforced concrete beam with 0.61% steel, subjected to a linear temperature drop of 55°C under the conditions of restraint mentioned in § 46. Fig. 8 showed the variation of the steel stress with time. Disregarding shrinkage and assuming $m=15$ in an elastic calculation, the steel stress should rise on heating to a value of 16,300 lb/sq. in. tension as shown. In fact, however, the maximum tensile stress achieved had been only about 7,000 lb/sq. in., and this had rapidly declined to zero; thereafter the steel went into *compression*, reaching a stress of 4,000 lb/sq. in. after 10 days from first heating. On cooling, after a further 10 days the steel stress had increased to 12,000 lb/sq. in. compression.

48. The results of this test differed so markedly from the calculated value that the experiment had been repeated a number of times with varying conditions, but the stresses recorded were in every case completely different from the predictions of elastic theory. A study of the problem suggested that the creep and shrinkage under the elevated temperatures were the cause of the apparently anomalous behaviour, and control tests on heated cylinders showed large creep and shrinkage values. The effects of moisture-loss appeared to dominate the problem and to dwarf thermal expansion and elastic strains. It should be emphasized that the concrete in these tests had a low water-content and therefore would experience a smaller moisture-loss than most concretes used in practice. The major influence of the moisture-loss had been confirmed in two ways.

49. First, a reinforced concrete beam had been subjected to a thermal test, as described above, but in this case moisture-loss had been prevented by sealing the whole specimen in a jacket of copper foil. Compared with a calculated steel stress of 9,500 lb/sq. in. tension for a temperature difference of 30°C, the recorded stress had risen on heating at the age of 80 days to 8,500 lb/sq. in. tension, and in a further 50 days had declined to 7,500 lb/sq. in. tension. Thus, the behaviour of a sealed specimen approached that of the elastic case. In a nuclear shield the conditions deep in the interior might be akin to the sealed case; while near the surface, moisture-loss would occur and would drastically affect the strains. The behaviour of the whole structure would therefore probably be intermediate between the extreme cases described above, and so a study of scale-effect and the penetration of drying into a heated mass seemed to be a vital topic for research. Incidentally, when the copper jacket covering the model beam had been cut away after 50 days of heating, the tension in the reinforcement had dropped to zero in 30 days, again indicating a severe departure from elastic behaviour; this was undoubtedly due to the release of moisture.

50. Secondly, some confirmation of the dominant role of moisture-loss had been obtained by a numerical analysis of the experiments. The depth of the beam had been divided into a number of small elements and the duration of the test into a number of intervals of time. Then, for each element the thermal expansion, elastic strain, shrinkage, and creep had been calculated by using data obtained from the control specimens; and at the end of each time-interval compatibility of strains had been secured by trial, thus yielding the stress distribution at the commencement of the next time-interval. This iterative procedure, which was readily adaptable to any condition of partial or complete restraint, was successful in predicting with reasonable accuracy the severe variations in stress typified in Fig. 8. The calculations showed clearly that shrinkage and creep had a predominant influence on the stresses and strains.

51. Professor Ross and Mr England appreciated that the Author had specifically excluded these factors. This was a perfectly proper restriction, at least in the first instance, in a complex problem in structural mechanics. Engineers were often forced to make assumptions that they knew to be untrue, in order to make a problem tractable. In this case, however, it seemed that the excluded factors might be more influential than those considered in the elastic analysis of the Paper; they certainly were so in small beams, and even in a nuclear shield it was probable that an elastic analysis, despite the use of a high modular ratio to allow for creep, would predict results far from the truth. It seemed important to make these points, since they might invalidate the discussion concerning the economics of a thermal shield and, indeed, the design of the reinforcement of the shield itself.

Mr D. J. Hannant (Research Student, Dept of Civil and Mechanical Engineering, University of Nottingham) observed that any analysis that assumed that concrete was a perfectly elastic solid which did not creep under stress, could not withstand any tensile stress, and did not shrink, was open to the criticism that the results were not valid in practice.

53. Initially accepting the assumptions made in the Paper, the 50°C temperature difference, obtained by the Author as a limiting figure with regard to the need for reinforcement in a slab, was solely that temperature difference below which the steel stress could not exceed 18,000 lb/sq. in., using the representative values quoted in the Paper. This temperature difference had virtually nothing to do with the necessity for the introduction of reinforcement. Where the reinforcement was needed only to control cracking, calculations showed that if in a 7-ft-thick shield no reinforcement was used at a temperature difference of 50°C, then probably no reinforcement was required at a temperature difference of 87°C, since the increase in crack depth was theoretically only 5.3 in. in approximately 70 in. Therefore an extra thermal shield was unnecessary. Also, if crack depth was to be limited by reinforcement, then the quantity needed to obtain the same theoretical crack depth at 50°C difference as at 87°C difference was certainly not nominal, but was approximately 80% of that required at 87°C difference.

54. Table 1 showed values calculated for a 7-ft-thick shield by using a linear temperature gradient. He would explain how these values had been derived. The notation, with the following exceptions, was the same as that used by the Author:—

- e_c denoted maximum permissible tensile strain in concrete
- t_c „ maximum permissible tensile stress in concrete
- V „ crack depth from cold surface
- T_V „ temperature at depth V above minimum as datum
- A_s „ area of steel per inch

The assumptions were the same as those in §§ 2–6 of the Paper. For simplicity of calculation the temperature gradient was taken as linear and the steel was assumed to be concentrated at the cool surface. The conditions to be considered were as follows:—

- (i) Unreinforced. General theory. Crack depth at $T_m = 50^\circ\text{C}$, and $T_m = 87^\circ\text{C}$.
- (ii) Reinforced. Particular case of crack depth at 87°C for simultaneous steel and concrete failure.
- (iii) Area of steel required in case (ii).
- (iv) Area of steel required to limit crack depth to that obtained in (ii), but with a temperature difference of 50°C.

Condition (i): Unreinforced slab (see Fig. 9)

Concrete cracked at depth V when: $\frac{\alpha(T_m - T_V)}{2} = e_c$ (30)

From similar triangles: $\frac{T_V}{V} = \frac{T_m}{l}$ (31)

FIG. 9.—STRAIN DIAGRAM FOR CONDITION (i)

Substitute (31) in (30):
$$\frac{\alpha T_m (l - V)}{2} = e_c \quad (32)$$

Rearranging:
$$V = \frac{l(\alpha T_m - 2e_c)}{\alpha T_m} \quad (33)$$

Let $t_c = 100$ lb/sq. in. $\therefore e_c = 0.443 \times 10^{-4}$ ($m = 13.3$, $E = 2.255 \times 10^6$ lb/sq. in.).
Let $l = 84$ in:

(a) $T_m = 50^\circ\text{C}$. On substitution in (33) $V = 71.6$ in.

(b) $T_m = 87^\circ\text{C}$. On substitution in (33) $V = 76.9$ in.

Condition (ii): Reinforced slab; simultaneous steel and concrete failure (see Fig. 10)

$$\text{Strain gradient} = \frac{\alpha T_m}{l} \quad (34)$$

$$\therefore l - V = \left(\alpha T_m - \frac{t_p}{mE} + e_c \right) \cdot \frac{l}{\alpha T_m} \quad (35)$$

FIG. 10.—STRAIN DIAGRAM FOR CONDITION (ii)

Rearranging:
$$V = \frac{l}{\alpha T_m} \cdot \left(\frac{t_p}{mE} - e_c \right) \quad (36)$$

Let $l = 84$ in., $T_m = 87^\circ\text{C}$, $e_c = 0.443 \times 10^{-4}$, $t_p = 18,000$ lb/sq. in., $mE = 30 \times 10^6$ lb/sq. in.
On substitution in (36): $V = 44.7$ in.

Condition (iii): Area of steel required in (ii)

Equating forces for unit width:

$$P_p \cdot \frac{z}{2} = A_s \cdot t_p + (l - z - V) \cdot \frac{t_c}{2} \quad (37)$$

$$z = 35.7 \text{ in.}, \quad P_p = 1,000 \text{ lb/sq. in.}$$

On substitution: $A_s = 0.981$ sq. in./in.

Condition (iv): Area of steel required to limit crack depth to 44.7 in. with a temperature difference of 50°C (see Fig. 11)

$$\text{Strain gradient} = \frac{\alpha T_m}{l}$$

From geometry:
$$l - z = V + \frac{e_c \cdot l}{\alpha T_m} = \frac{t}{mE} \cdot \frac{l}{\alpha T_m} \quad (38)$$

$$V = 44.7 \text{ in.}, \quad T_m = 50^\circ\text{C}, \quad l = 84 \text{ in.}, \quad e_c = 0.443 \times 10^{-4}$$

Substitution in (38) gives: $t = 10,900$ lb/sq. in.

$$z = 33.1 \text{ in.}$$

$$P_{\max} = 533 \text{ lb/sq. in.}$$

FIG. 11.—STRAIN DIAGRAM FOR CONDITION (iv)

Substituting the appropriate values in equation (37): $A_s = 0.781$ sq. in/in.

TABLE 1

Temperature difference T_m : °C	Concrete tensile strength t_c : lb/sq. in.	Crack depth V : inches	Steel area A_s : sq. in/in.
50	100	71.6	0
87	100	76.9	0
50	100	44.7	0.781
87	100	44.7	0.981

55. The Author had made the following assumptions:—

- (a) Shrinkage stresses were neglected.
- (b) Concrete was a perfectly elastic solid which did not creep under stress.
- (c) Concrete could not withstand any tensile stress.

Mr Hannant then considered the effects of these assumptions on the steel stresses, and indicated the results of tests carried out at Nottingham University.

56. *Shrinkage*.—A value of 6×10^{-4} in/in. total shrinkage was not unusual for ordinary Portland cement concrete. This was equivalent to a maximum tensile-stress loss in the steel of 18,000 lb/sq. in., the actual value depending on the percentage of reinforcement.

57. *Creep*.—The combined effects of compressive, tensile, and shear creep of the concrete were difficult to predict, but they would probably reduce the tensile stresses in the steel while the section was hot. Creep in tension would reduce the theoretical crack depths.

58. *Tensile strength*.—The tensile strength of concrete, although generally neglected, could exert considerable force when a large area was entailed, as in a biological shield. In practice, it might substantially reduce the design tensile stresses of the steel.

59. Tests had shown that both creep of the concrete in tension and shrinkage were accelerated by increase in temperature, particularly at temperatures above 80°C.

60. Measurements had been made of steel stresses in reinforced concrete blocks

subjected to linear temperature gradients. A summary of the results obtained was as follows:—

- (a) When heating was sufficiently rapid to ensure that shrinkage and creep had little effect, the measured steel stresses were in close agreement with the values calculated from the elastic analysis and the assumption that concrete had a tensile strength of $\frac{1}{10}$ of the compressive strength.
- (b) After a period at continuous high-temperature gradients, the tensile reinforcement developed compressive stress even while the section was hot. An average compressive stress of 4,500 lb/sq. in. was measured at a temperature difference of 85°C, while the theoretical stress was 19,600 lb/sq. in. tension. When cool, residual compressive stress of 10,000 lb/sq. in. was measured in the tensile reinforcement. Under these conditions the effect of the tensile reinforcement was to increase the concrete tensile stress and to act as a crack former.
- (c) The increase in rate of shrinkage of the concrete at high-temperature gradients appeared to be the major factor in the reduction of the steel tensile stress.
- (d) A 25% reduction in concrete thermal conductivity was observed during the tests and appeared to be related to the shrinkage. The result was to give an increasing temperature gradient for a given heat input.

The concrete blocks under test measured 3 ft × 2 ft × 1 ft 5 in., weighed approximately $\frac{1}{4}$ ton, and had a plane of heating wires 3 ft × 2 ft down the centre.

61. It was known that the effect of the size of the concrete mass on the rate of drying shrinkage of concrete was considerable, and it was therefore not to be expected that a shield 7 ft thick would give similar compressive steel stresses in as short a time as the blocks tested. However, there were indications that the tested concrete dried out from the centre (i.e. the position of highest temperature) outwards, which was the opposite of the normal drying distribution. This, together with the accelerated nature of the shrinkage due to high temperatures, would seem to be a considerable argument in favour of including in the design of biological shields the loss of steel tensile stress caused by shrinkage.

Mr I. T. Ellis (Senior Assistant Engineer, Research and Development Dept, Central Electricity Generating Board) thought it unfortunate that the structural effect of thermal load on reinforced concrete was usually assessed in terms of stress. Since thermal stresses arose only when thermal strains were restrained, it would be far more logical to assess the effect of thermal load in terms of strain. The limiting factor would then be maximum permissible strain and not maximum permissible stress. He would show (§ 68) that the two criteria were not necessarily synonymous, even within the elastic limits of the materials.

63. It was possible to illustrate the advantage of using strain as the design criterion by considering again some of the examples given in the Paper, but first it was pertinent to review some of the assumptions that had been made.

64. Whereas it was necessary to make simplifying assumptions before attempting to consider analytically the effects of thermal load on concrete, these assumptions must be borne in mind when drawing economic conclusions from the work. For instance, as mentioned in § 62, thermal stresses arose when thermal strains were restrained, and therefore the fact that concrete crept under stress was of considerable importance. Again, the apparent coefficient of expansion of concrete might decrease appreciably with temperature and invalidate any analytical conclusions that had been drawn. The assumption made in § 6, that there must be some reinforcement in the plane of maximum concrete tensile strain, was not valid; in fact, it would be physically impossible to comply with this requirement, since the steel reinforcement must have some cover. Mr Ellis would show (§ 68) that moving the steel in slightly from the surface had a profound effect on the thermal stress pattern. It must also be remembered that there was no

need, from the shielding aspect, to work to unusually high standards of crack control⁵.

65. If in the example given in § 19 the maximum stress in the steel, t_p , was assumed to be 30,000 lb/sq. in. (C.P.114, 1957),

then: $T_m' = 120^\circ\text{C}$, with $P_p = 1,000$ lb/sq. in.

or: $T_m' = 83^\circ\text{C}$, with $P_p = 0$.

If t_p was assumed to be 160,000 lb/sq. in. (using high-tensile prestressing wire for crack control),

then: $T_m' = 480^\circ\text{C}$, with $P_p = 1,000$ lb/sq. in.

or: $T_m' = 445^\circ\text{C}$, with $P_p = 0$.

66. Since the physical properties of ordinary Portland cement concrete deteriorated rapidly above 200°C , a temperature difference of the order of 150°C was the maximum that could be contemplated. Therefore, the second case given above was obviously not feasible, but it did illustrate the order of temperature differences that might be tolerated theoretically, while working within accepted stress limits. Equating $P_p = 0$ in the cases above implied, of course, that the concrete section was cracked over its full depth.

67. Since, as was shown in § 65, it was possible to keep within permissible stress levels, the economics of a thermal shield versus reinforcement, which was discussed in §§ 30-37, should be assessed in terms of maximum permissible strain. When, as in the case of the top of the biological shield, the section had also to carry considerable structural loads, greater care had to be exercised and each case must be considered on its merits. It was possible that the maximum permissible temperature difference would be determined by the maximum permissible deflexion of the top slab, and not from a consideration of the deterioration of the properties of concrete, or from the biological-shielding aspect.

68. If, in the case described in § 43, the steel was moved in 6 in. instead of being, as assumed, concentrated at the under surface, then for the "hot" case the lever arm would be reduced from 114.8 in. to 102 in., instead of to 33 in. The steel area required for the "hot" case was therefore 0.096 sq. in./in. compared to 0.0853 sq. in./in. for the "cold" case, and to 0.296 sq. in./in. for the "hot" case, if the steel was concentrated at the under surface. Both of the reinforcement arrangements discussed above were structurally satisfactory, the only difference being that when the steel was moved in 6 in. the steel required was considerably reduced, but it did mean an appreciable increase in maximum strain, as could be seen from Fig. 12. Again this illustrated that it was far more logical to use maximum permissible strain as the design criterion.

69. Since there was only limited experience on the effects of a variable temperature distribution across a reinforced concrete section except for small temperature differences, it would be necessary to verify experimentally that the type and arrangement of reinforcement proposed in the design was satisfactory, and able to control and keep within permissible levels the cracking due to the variable temperature distribution.

The Author, in reply, thanked Professor Ross and Mr England for their account of their interesting and useful experiments on small reinforced concrete beams. He felt, however, that the effect of scale, which they had mentioned, had not been given its proper significance. For instance, Spencer⁶ had measured a water loss of only 10% after 10 months at a depth of 11 in. from the surface. Obviously, therefore, the overall shrinkage in a shield about 10 ft thick might be quite small.

71. In reply to Mr Hannant, the Author was confident that he was not alone in feeling strongly that in order to be certain of obtaining a shield that had no wide cracks

⁵ B. T. Price, C. C. Horton, and K. T. Spinney, "Radiation shielding". Pergamon Press, London, 1957.

⁶ R. W. Spencer, "Measurements of the moisture content of concrete". Proc. Amer. Concr. Inst., vol. 34 (1937), p. 45.

FIG. 12

due to surface shrinkage and temperature differences there must be some reinforcement near the cold surface of the shield, and this reinforcement must not be too highly stressed.

72. Mr Hannant's observations on the *probable* effects of creep and tensile strength of the concrete, although quite sound, would carry little weight with the practical shield designer until "probable" could be replaced by "certain".

73. Mr Ellis had not made clear how the criterion of maximum permissible strain would be applied and how the results would differ from those obtained by using a maximum possible stress. For instance, if in § 65 one worked with a maximum permissible strain corresponding to a stress of 160,000 lb/sq. in., then exactly the same maximum permissible temperature would result. The increase in maximum strain shown in Fig. 12 was mostly due to the fact that total strain had been plotted. Should it then be assumed that the criterion was maximum total strain? This would be unreasonable, since in the side shield where there was a constraint which kept the slab flat there could be large stresses with zero total strain, the strain due to stress cancelling the temperature strain.