Paper No. 6391

An experimental study of a model tied-arch bridge†

by

William George Godden, M.Sc., Ph.D., A.M.I.C.E.

and

James Crawford Thompson, M.Sc., A.M.I.C.E.*Discussion*

Mr Chin Fung Kee (Senior Lecturer, Department of Engineering, University of Malaya) observed that in the design considerations the Authors had omitted a very important point in the design of the arch ribs, viz., the unavoidable eccentricities. The theoretical value of H_G and its value obtained by extrapolation by applying Southwell's method, however, gave the elastic buckling load of an *ideal* or *perfect* arch. In practice there would be eccentricities due to imperfect construction and alignment, non-axial application of the horizontal thrust, non-homogeneity of the material, and lateral curvature resulting from a transverse wind load. Those eccentricities had to be taken into consideration in assessing the maximum load that the structure could carry. In an actual structure, owing to the unavoidable eccentricities the arches would deflect laterally immediately on the application of a load, and the elastic buckling load H_G was never reached because of the superposition of the stresses due to bending and to torsion.

26. If the Southwell plot was produced, the intercept e on the Z -axis represented the combined effect of the eccentricities. If the deflected form was of buckling mode 1, its equation was:

$$Z = k(\sin \pi x/L - \frac{1}{3} \sin 3\pi x/L) \quad \dots \dots \dots (1)$$

where k was a constant. Then at the crown of the arch where $x=L/2$,

$$k = \frac{3}{2}z \quad \dots \dots \dots (2)$$

The maximum fibre stress occurred at the cross-section at the crown of the arch rib on the concave side, and was made up of⁷:

- (a) the average compressive stress $f_a = H/A$, where A was the cross-sectional area of the arch rib; and
- (b) a compressive stress f_{bc} due to a bending moment equal to $0.4142 kH$.

There was no torque at the crown.

27. Although the deflected form under a uniform lateral load was only an approximation of buckling mode 1, it had been shown⁸ in experiments in which the arches were loaded to failure that there was close agreement between the collapse load registered and the maximum load calculated by the method suggested in § 26. Indeed, even in the experiments where the lateral load was in the form of a single point load applied at the crown of the arch rib, the maximum load that the structure could sustain under test was in close agreement with the calculated value; in no case was the difference greater than 4%.

28. The small effect that a departure of the deflected form from the characteristic buckling mode had on the value of the critical load, had been proved by Southwell⁹ in the case of the pin-ended strut. Both Dr Godden⁴ and Mr Chin Fung Kee had found

† Proc. Instn civ. Engrs, vol. 14, p. 383 (Dec. 1959)

References 7 *et seq.* are given on p. 246.

by experiment that in the case of the tied arch, lack of symmetry and deviation from the characteristic buckling mode had no apparent effect on the crippling load assessed from the test results. Therefore, if the difference between the value of the crippling load when the arch was under an applied lateral load, and that computed on the basis that the buckling mode was given by equation (1), was small, then the error in assuming that the expressions for bending moment and for torque for the arch under the applied lateral load were the same as those derived for an arch deflecting under the characteristic buckling mode was even smaller. That was so because the theoretical value of H_{cr} was obtained on the energy criterion of stability, and the total strain energy was derived by integrating the square of the expressions for bending moment and for torque.

29. The expressions (a) and (b) in § 26 would obviously not apply for arches deflecting under buckling mode 2. But structural design was normally concerned with the determination of the lower limit of the load capacity of a given structure after consideration of all the different systems of loading that might possibly be imposed on the structure in practice. In the case of the tied arch, the second buckling load was higher than the first. Experiment had shown that it required the application of only a comparatively small constant lateral load to eradicate any tendency of the arch to assume the second buckling mode. In practice, therefore, under the full design transverse wind loading, the transverse deformation of the arch rib would conform more to the first buckling mode. It was interesting to note from Fig. 6 that the intercepts e on the Z -axis of the Southwell plots for arches A1 and A2 were nearly equal in value. It was significant that it required nearly twice the static wind loading on arch A1, which showed the tendency to deform into buckling mode 2, to produce the same value of e . It would therefore appear that the maximum load that the arch could carry as calculated by the method suggested in § 26 would still be the lower limit of the carrying capacity of the arch, even if it had the tendency to fall into the second buckling mode at small values of H when under a small or zero transverse load. Indeed, Mr Chin Fung Kee had found that to be the case with arches loaded to failure.

30. From Fig. 6 the value of e was found to be nearly 0.106 in. The ideal collapse load was 1.06 ton, thereby giving a maximum stress of 5.8 tons/sq. in. if the arch ribs were without any imperfection. For the purpose of comparison a maximum stress of 5.8 tons/sq. in. was also used to evaluate the maximum load that the arches A1 and A2 could each carry, taking into consideration their eccentricities. It could be shown that that maximum load was 0.83 ton. There was, therefore, a reduction of nearly 22%. Even using the proof stress of 6.5 tons/sq. in. as the criterion, the maximum load was 0.854 ton, which was only about 80% of the ideal buckling load—and this was based on the eccentricities of a laboratory model. The imperfections of the parent structure could be greater. The design load must therefore relate to the eccentricities and imperfections of the structure concerned, and the method of obtaining the design load by simply dividing the ideal collapse load by an arbitrary factor was not only unsound but might result in serious error.

31. It was most interesting that in Fig. 6 there was no suggestion of non-linear behaviour in the system, even though the values of H were less than $0.5H_{cr}$. Mr Chin Fung Kee had also recorded similar behaviour for single arches. That was evidence that in this respect the tied arch did not behave like the pin-ended strut for which structure, Timoshenko¹⁰ had proved that for small values of H the experimental results would not fall on a straight line.

The Authors, in reply, stated that they were aware of the eccentricity and transverse loading problem, and also of the experimental work done by Mr Chin Fung Kee on it. There had, however, not been a general analytical solution available for this problem, nor had any experimental work been carried out that included actual strain measurement.

33. Before any stress analysis was applied to the results given in the Paper with a view to predicting collapse by overstrain, the following points should be noted:—

- (i) Eccentricity errors would obviously be greater in a model formed in this manner than on the parent structure. Hence, the test results shown in Fig. 5 were

not intended to be taken quantitatively (see § 15). The Authors could not agree that the 0.1 in. eccentricity found in the alignment of arch A2 represented to scale a realistic eccentricity in the construction of the parent structure; alignment was one of the problems of experimental work of this kind.

- (ii) The transverse loadings used to produce the results of Fig. 6 represented respectively about 2.8 and 1.4 times a static wind load of 30 lb/sq. ft on the parent structure (see § 19). Not only were the initial eccentricities disproportionate, but the transverse loading applied for the purpose of finding the elastic stability value represented a 40% overload in A2.

34. Care was needed in applying strain criteria to a model specifically designed to study elastic behaviour—alignment was different, and material properties were different—and the Authors did not consider that § 30 in Mr Chin Fung Kee's contribution was relevant to the original design problem.

35. Within the range of elastic action, the problem of transverse collapse was a quite ordinary elastic stability problem, liable as much as any such problem to scatter in the Southwell plot at low values of H , the degree of scatter (as for the pin-ended strut) depending on the form of the initial eccentricity that caused deflexion. In this respect extensive experimental study showed that no basic difference existed between the transverse buckling of an arch rib and the buckling of a pin-ended strut as suggested by Mr Chin Fung Kee in § 31, though the stiffening effect of the tension membrane did complicate the analysis.

36. Any conception of the effect of transverse wind loading must include all the factors, including the torsional stiffness of the arch rib. The total stress was partly due to axial thrust, and partly due to bending; the latter was made up of two components, namely the simple effect of wind on its own, and added to that, the effect of H acting on the total eccentricity. In order to study the last of these, namely the moment due to H and the total eccentricity, there must be some knowledge of alignment errors and also of the deflexion caused by wind; this must be related to the original structure, of course, though the model could be used to verify the accuracy of any analytical method applied to the arch rib on its own, and further, how that was related to the failure of the entire structure. A general solution of the problem of stress and deflexion due to axial and transverse load in an isolated arch rib had been made, and it was hoped, would be published shortly.

37. It should be noted that collapse by overstrain was not necessarily the ruling criterion (reference 4 described a model arch supporting its full critical load as found by elastic stability); it was quite possible to get elastic stability failure, and obviously both factors must be kept in mind. As was stated in § 9, a factor of 2.5 had been chosen in the steel design against failure by transverse elastic instability (to have made it any higher would have made an elastic stability study almost impossible), and this was intended to cover any small reduction in load-carrying capacity by the transverse loading—even with the 20% reduction suggested by Mr Chin Fung Kee for the model there was still a factor of 2.0. Recent study suggested that the design thus produced would not need to be greatly modified even when the static effect of wind load was taken into account, and of course the factors influencing stability were exactly as stated; but to vindicate a theory relating to inelastic action or overstrain, a destructive test would need to be carried out on the complete structure.

REFERENCES

7. Chin Fung Kee, "The design of the unbraced Stabbogen arch". *Struct. Engrg*, vol. 37 (1959), p. 265.
8. Chin Fung Kee, "The inelastic buckling of arches". M.Sc. thesis, Queen's University, Belfast, 1953.
9. R. V. Southwell, "An introduction to the theory of elasticity for engineers and physicists". 2nd edn. Oxford University Press, Oxford, 1941.
10. S. Timoshenko, "Theory of elastic stability". McGraw-Hill, New York, 1936.

Paper No. 6423

On stability of two-dimensional structural frameworks†

by

Hsuan-Loh Su, M.Sc.Tech.*Discussion*

Dr Arthur Bolton (Senior Lecturer in Civil Engineering, University of Liverpool) observed that the Paper gave a wrong impression of current thought and contained errors of computation.

74. Authors of recent Papers were usually well aware of the difference between the elastic critical load and the failure load of a structure, and of the limitations of the elastic critical load itself. A great deal of work had been carried out to obtain a method of calculating the failure load of structures, but it was not to be expected that such a difficult calculation, complicated by so many unknowns and with such assorted non-linearities, could be carried out in one step. For that reason many workers, although they had the calculation of failure loads as their objective, were content to improve individual parts of the whole calculation. A great deal of progress in the computation of elastic critical loads had been made in the past few years. Since this kind of approach had often been explained^{16, 17}, and since failure loads had been successfully predicted by calculations¹⁸ using the elastic critical load as one parameter, it was not true to claim that the calculation of critical loads for frameworks subject to axial loads only was essentially a mathematician's recreation (§ 31). Indeed, that calculation seemed to offer the only practicable way of obtaining failure loads of practical structures.

75. In §§ 26 and 27 a clear explanation of individual member collapse and frame instability was not given. As a first step it was necessary to specify the kind of frame exactly. In the pin-jointed perfect frame it was possible to have a collapse involving only one member. Real frames, however, usually had redundancies, owing to gusset-plated or rigid joints and possibly redundant members, and the whole frame was involved in the elastic critical load and hence in the failure load. Very often some small part of a framework would be isolated by plastic hinges before total failure occurred, but even then the failure load depended on the initial elastic behaviour. That was one of the reasons why a formula like equation (4) offered no hope of a rational design method—no bending moment was known without an analysis that was far too complicated to be practicable.

76. In § 27 the physical criterion of instability was defined as the "bifurcation of the equilibrium position". That was a mathematical test simple to apply to isolated members, but containing pitfalls which might lead to errors in more complicated structures. A definition of critical load free from that difficulty was that the critical load of a structure was the smallest load at which the structure became unable to resist every infinitesimal load which could be used as a test. The latter definition drew attention to the fact that to test the stiffness of an arbitrarily chosen joint, or the indeterminacy of an arbitrarily chosen deflexion, was not sufficient.

77. With regard to the method of calculation of elastic critical loads shown in the Paper, there were two sources of error. The first was in the treatment of sway and the second in the treatment of triangulated frames.

† Proc. Instn civ. Engrs, vol. 16, p. 143 (June 1960).

78. The “no-shear” coefficients n and o in § 39 involved sway of the stanchions. Hence, when joint B was rotated it also deflected, and sway moments were caused on stanchion CD since it was compelled to sway by the beam BC. Moments were transmitted to joint C by the sway as well as by carry-over across the beam. Hence, all the coefficients of equation (5) were in error, and the critical load was at $0.745 P_E$ instead of the value $0.531 P_E$ given in the Paper. On the other hand, the calculation for the multi-storey frame in § 52 ignored the carry-over across beams. The comments in §§ 42, 43, 44, 59, and 63, based on these errors, were unjustified.

79. In the case of Example 3 the calculation of § 48 had two errors of principle. First, the signs of the joint rotations had been ignored, and secondly, the right-hand half of the frame had been disregarded. If joint B were rotated clockwise the adjacent joints A, D, E, and C would not all rotate in the same direction. Moments transferred to A and back to B returned with a sign different from those transferred from A to D and thence to B. When the right-hand half of the frame was ignored the transmission factors from C and E were only half of what they should have been. If account had been taken of signs and the other half of the frame, the value of Q_B would have been found to be equal to unity, so that the comments of §§ 48, 49, and 50 were again unjustified.

80. It was of interest that the particular example demonstrated had already been calculated by the method advocated by the Author. First^{19, 20}, only the four major terms (adding to 0.720 in § 48) had been used as an approximation, and in a second Paper^{20, 21} all the terms had been considered—including even second-order terms omitted by the Author of the present Paper.

The Author, in reply, stated that the main theme of the arguments in the discussion was unmistakably Eulerian, as could be expected at the present stage of development. It was difficult to establish a non-Euler approach within the space available. Since this concept would be explored in a future Paper, a piecemeal reply seemed appropriate.

82. The implication of the failure load mentioned in § 74 was rather vague. From the emphatic attachment of the adjective “elastic” to the critical load, it seemed that this failure was implied to differ from the collapse associated with the plastic methods. However, if the critical load were defined as the smallest load at which the structure became unable to resist any infinitesimal load (§ 76), then there would be little distinction between the critical and the collapse load. The zero-stiffness concept and the deflexion indeterminacy concept were at least consistent. Whether they were “sufficient” could not be determined by logomachy alone.

83. A great deal of progress in the computation of critical loads at the mechanistic level had indeed been made, notably by Merchant and his team in the past few years. Nevertheless, the only reaction system investigated was the unclad framework and the only action system considered was a simple pattern of axial loads which resembled the unrealistic proportional loading in the plastic method. Structural reality still seemed far from such a Utopia and there was little ground for complacency. It was precisely with such a situation in view that the amplification method was employed for solving stability problems which were too complicated to tackle otherwise.

84. From the title of the Paper, it was apparent that only comments on the current situation were offered. No effort had been made to establish a comprehensive method based upon the non-Euler concept. Some misunderstanding seemed inevitable. For instance, equation (4) was the type of hypothesis often adopted by the Euler school, though its limited scope of applicability was well known. However, it was untrue that no bending moment could be found without an analysis that was far too complicated to be practicable. The problem here was what method to use. In fact, a numerical method and a computer programme for such analyses would be available soon.

85. The “bifurcation of the equilibrium position”, mentioned in § 27, was the simplest but by no means the most satisfactory physical criterion of instability (obviously not a mathematical test). On the other hand, the neo-plastic or neo-collapse definition given

in § 76 seemed rather evasive. There were various combinations of geometrically different disturbing systems which could make a real structure unable to resist any further infinitesimal loading. The smallest load did not exist in reality, and a stability test could grow out of its "recreation" stage when and only when the real structural behaviour was seriously taken into account.

86. It was unnecessary to glorify "frame instability" and to elevate it to a much higher level than that occupied by the "individual member collapse". The purpose of any investigation on framework stability was to find the relation between the behaviour of the frame and that of its constituent members when axial loads played a prominent part. As a matter of fact, the behaviour of individual members was of much more direct interest to the designer.

87. The coefficients in equation (5) were approximate expressions. Their exact forms involved some stability functions which were not explicitly tabulated, and generally necessitated a solution of a transcendental equation. (The solution of the exact case gave a value of $2.552P_E$ for the symmetric and $0.747P_E$ for the asymmetric case, corresponding to two roots of the transcendental equation.) These coefficients would be exact when D was allowed to displace horizontally, and one of the roots, $n=6$, would give $0.747P_E$ for the asymmetric case. It should be noted that a solution based on the present method would provide two roots corresponding to the symmetric and the asymmetric case, whereas previous methods could only tackle one case at a time.

88. As to the carry-over across beams in Example 4, it was useful to consult reference 7, which would show that such carry-over had been considered. This anti-symmetric case was not the most general but was adopted merely for illustration. A similar manoeuvre to that applied to Example 2 would again produce two roots representing both the symmetric and the asymmetric case.

89. There was a slip in the numerical operation to assess Q_B in Example 3, which was not an error of principle. The approximate value of Q_B should be:

$$(0.414 + 0.008 + 0.275 + 0.275 + 0.022 + 0.022) - (0.043) = 0.973$$

which gave $G_B = 37$. This was a large value. Nevertheless, it was small in comparison with infinity. A calculation incorporating bending effects would reveal that an amplification factor smaller than 10 was sufficient to cause an instability collapse. This again showed that the infinity or the zero concept could only lead to a weak criterion.

90. In Example 3, if the right-hand half of the frame were disregarded, the transfer factors from C and E would be roughly doubled but not halved. It should be pointed out that the sign of the transfer factor was inherently negative as clearly expressed in §§ 15 and 39. This was a topological property of any prismatic structural element and did not depend upon whether the frame was triangular. Only in an electric or a hydraulic network which might have the same topological appearance as a structural network, could the transfer factor be inherently positive.

91. The comments mentioned in §§ 78 and 79 were, apparently, not based upon the examples. On the contrary, the examples were used to illustrate comments. It would contradict logic if methodologic reasoning could be used to discredit philosophical conception.

92. The amplification method suggested in the Paper was stated to have been used previously to solve Example 3. However, the difference between the present method and that in references 19 and 20 was obvious, from many aspects.

93. First and foremost, the amplification principle, as the relaxation or the iteration principle, was a method of numerical analysis in its own right. This principle had been used in the Paper to develop a method of general structural analysis, and could be applied to both Euler and non-Euler cases, whereas other methods were merely a derivative of existing methods of framework analysis, and could only be used in Eulerian problems.

94. So far as methodology was concerned, the present method, focusing on the reaction characteristics of the framework, was superior to various convergence methods in which the action and the reaction were blended in computation, and consequently

complication and confusion frequently arose. Convergence was a derived mathematical property while those reaction characteristics were fundamental and directly conjoined with physical reality.

95. It was difficult to delve into all the minor points of the method suggested in references 19 and 20. So far as the triangular truss was concerned, it was evident that not only the second-order terms but also the first-order ones had been neglected if only four out of six major terms had been used (§ 80).

REFERENCES

16. W. Merchant, "The failure load of rigid jointed frameworks as influenced by stability". *Struct. Engr*, vol. 32 (1954), p. 185.
17. Discussion to reference 21. *Struct. Engr*, vol. 38 (1960), p. 239.
18. W. Merchant, C. A. Rashid, A. Bolton, and A. Salem, "The behaviour of unclad frames". *Proc. 50th Anniv. Conf., Instn struct. Engrs*, Oct. 1958. See p. 163.
19. A. Bolton, "A quick approximation to the critical load of rigidly jointed trusses". *Struct. Engr*, vol. 33 (1955), p. 90.
20. A. Bolton, "Structural frameworks". Ph.D. Thesis, Manchester Univ., 1957.
21. A. Bolton, "A convergence technique for determining the elastic critical load of rigidly jointed trusses". *Struct. Engr*, vol. 37 (1959), p. 233.