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Plastic collapse loads for circular-arc bow girders†

by

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and

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Dr M. R. Horne (Lecturer, Department of Engineering, University of Cambridge) observed that the Authors had presented very clearly the theory of the plastic behaviour of rings bent out of their plane. He described a graphical treatment which to some extent simplified the problem for a concentrated load.

60. AB in Fig. 20a represented an element of a bow girder; the element was subjected to a bending moment N about some axis EF. The line CD was normal to the axis of the element, and the angle between CD and the axis EF of the bending moment N was θ . The values of N causing complete plasticity were plotted against θ in the polar diagram in Fig. 20b. If equation (2) (p. 162) represented the plastic behaviour of the element, then the polar diagram was an ellipse with principal axes of half-length M_p ($=T_p/\alpha$) and T_p as shown. The axis of deformation of the element due to a bending moment represented by point a on the ellipse was parallel to the tangent gh, thus giving the axis of deformation GH in Fig. 20a.

61. Consideration should then be given to an element at any point A in a circular-arc bow girder of radius r , as shown in Fig. 21a. An ellipse was drawn in the plane of the girder, the principal axes CD and EF being respectively tangential and normal to the axis of the girder at A; $AC=s/\alpha$ and $AE=s$. Then the ellipse represented the possible positions of a resultant force of magnitude $Q=T_p/s$ which, if it acted on the girder at A, would just produce full plasticity. By drawing a similar ellipse round another element in the girder at B, it was immediately apparent whether a resultant force Q at any particular position where it just produced plasticity at A would be within the carrying capacity of the girder at B. Thus a force Q applied at G would produce full plasticity at A but not at B. A force Q could not, however, be applied at H without the carrying capacity at B being exceeded.

62. If the resultant force acting on a girder was constant over a considerable length, as when concentrated loads only were applied, it was convenient in solving problems to retain the same ellipse, but to move instead the position of the resultant force relative to the ellipse. Thus, to find the effect of a total force Q acting at a distance s from a girder JK of radius r (Fig. 21b), imagine the position of the force to move on the arc of a circle of radius $r+s$ (MN in Fig. 21c). If the arc MN in Fig. 21c lay wholly within the ellipse representing the plastic strength of the girder, then the girder JK in Fig. 21c would not fail owing to the force Q acting at I. If MLN in Fig. 21c just touched the ellipse at L as shown, but otherwise lay inside the ellipse, then the force Q in Fig. 21b would produce a plastic hinge at V but not elsewhere. For that to be possible, the curvature of MLN must be greater than that of the ellipse at L. Hence it followed that $s > \frac{\alpha^2 r}{1 - \alpha^2}$. That condition agreed with that stipulated by the Authors in § 10, rule (b).

63. Dr Horne then used the ellipse to solve case 1 for the loading of a bow girder

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FIG. 21.—GRAPHICAL TREATMENT OF PLASTICITY CONDITION FOR POINT LOAD IN CIRCULAR ARC

The criterion used by the Authors in relation to the intersection of the axes of bending for the three hinges was again used for case 2. Since the analytical solution for case 3 was elementary, the graphical approach had no advantage.

Dr E. H. Brown and Dr A. Gent (Civil Engineering Department, Imperial College) noted that the Paper anticipated by a few months the publication of very similar work which was in progress at Imperial College. They were especially pleased to see a discussion of the approximate yield criterion (equation 2 of the Paper). That seemed basic to the solution, and it was not at all obvious that the criterion adopted must be a good approximation.

67. A great part of the difficulty lay in the definition of T_p , the “full plastic moment in pure torsion”. Pure, and constant, torsion of a straight bar would produce plasticity throughout its length, and in general plastic warping of the cross-section would occur. Pure torsion of a bow girder, on the other hand, could occur only at an isolated cross-section where there happened to be no bending moment, and if this section was fully plastic it would lie between lengths of beam which at most were only partially plastic.

68. It was well established that during plastic collapse no changes occurred to the elastic strains, so that non-plastic parts of the structure became virtually rigid; consequently, plastic warping of the isolated cross-section would be restricted if not entirely prevented. The purely torsional moment required to give a fully plastic section should

It followed, therefore, that the torque must slowly increase after attaining the value T_p , as the twist increased.

72. In the tests on the H-section described in the Paper, a rapid increase in deflexion occurred at a load within a few per cent of the load calculated on the assumption that T_p was the fully-plastic torque with unrestrained warp. Those tests seemed to show, therefore, that the restraint on the warp at a hinge in pure torsion did not invalidate the practical estimation of the collapse load as described in Part I of the Paper. More theoretical and experimental work was, however, needed to explain the behaviour of this type of plastic hinge and the results of the investigations of Dr Brown and Dr Gent were awaited with much interest.

73. The effect of restraining the warp was shown in some recent tests carried out in the Department of Civil Engineering, University of Sheffield. Straight lengths of the H-section, described in § 29, were tested in torsion and also in combined bending and torsion. In these tests the centre section of the members was maintained plane, but free warp was permitted at the ends as far as possible. For a length $l=1$ ft between the supports for the specimen, the value of T_p was 48% greater than that determined in a standard torsion testing machine which allowed unrestricted warping. When l was 2 ft T_p was 26% greater than for the standard torsion test.

74. The tests in combined bending and torsion showed that equation (2) gave a true bound curve to the experimental points.