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A study of the movement of ships subjected to wave action†

by

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Professor A. L. L. Baker (Professor of Concrete Technology, Imperial College of Science and Technology) thought that the principal new factor which the Paper had exposed was the relation between the characteristics of the fender and the ranging of the ship under certain wave action. For some time now fenders had been designed with a certain kinetic energy of shock absorption to deal with a ship colliding at first berthing or when leaving the berth, and the force of impact at the time of such collision had also been considered—though it had often been overlooked—and fenders with a long stroke had been advocated. Attempts had been made to bring the force of impact at the time of collision down to a value which was reasonable for a tanker's hull. The Author, however, had introduced a new factor and had shown very clearly that the resilience of the fender largely determined the speed with which the ship ranged when it was tied in the berth.

91. The Author had shown from his results that a fender with a greater movement (about 5 times) could reduce the force of impact to $1/20$ th, which was very significant in relation to the build-up effect that fenders had with only a short movement, such as with a rubber fender with only 14 in. of movement.

92. The Author stated that in the berth which had been reproduced in the model there had been serious damage to ships' plates. That was not surprising, in view of the magnitude of the impact forces to which he referred; he had shown that it was possible to get impact forces of as much as 800 tons. Professor Baker thought that in reality that would probably be a little high, because ships' plates would start buckling and the frames would start bending, so that the ship would deflect a little and provide some resilience, which would reduce the force to a certain extent; but damage to the plating was a serious matter and it emphasized the importance of providing adequate resilience.

93. For many years Professor Baker had been trying to find a good criterion for the safe pressure which could be put on the side of a tanker when it was at a berth. The nearest figures that he had been able to obtain were deductions from Lloyds Design Tables, and, looking at those Tables, the conclusion would be reached that 30–40 tons was about the maximum concentrated pressure which could be put on the side of a tanker. He would like to advocate once more in the Institution a joint meeting between harbour engineers and naval architects with a view to determining more precisely this important criterion of design. In designing a bridge which cost only £10,000 the engineer would study the loading very carefully. Surely it was much more important to study carefully the loading on the side of a tanker which might cost more than £1 million. It seemed extraordinary that at the present stage of engineering development it was not possible to give a very precise value to that figure, and it was worse still to consider how many jetties had been designed without that figure being considered in the design at all, but only the kinetic energy of the vessel which had to be absorbed. It would be very interesting to make a survey of the work carried out in dry docks in Britain. He felt sure it would be found that tremendous expense was incurred annually in repairing plates which had been damaged about the waterline level in vessels due to the inadequate design of fendering systems.

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94. He had been unable to find in the Paper any reference to the influence of the position of the ship in relation to the fenders. If the moorings were moved further and further out, a stage would eventually be reached at which the ranging would just bring the ship into contact only with the fendering. There might be cases in which a good way of berthing would be to berth the tanker a little off the jetty and let it range in and out, and then the stiffer type of fendering would probably not cause build-up of ranging and do quite so much damage.

95. Did the weight of the fender have a very great influence? Some gravity fenders were much too heavy, and by being too heavy they possibly acted to some extent in the same way as rubber fendering and caused increased build-up of ranging and rolling of the ship.

96. How had the Author calculated the wind force of 30 tons on the side of the tanker with a wind of 30 knots? At present there was great apprehension about the effect of wind force on super-tankers in berth. There was a tendency to overestimate the wind force. Had the Author discovered any experimental or other evidence, such as stress on mooring ropes, which would throw some light on that problem?

97. Finally, Professor Baker showed a short length of film of a tanker in a berth in the north-east. The ship was ranging against horizontal gravity fenders of the open-frame type. The fenders weighed about 25 tons and had a movement of about 45 in. When the tanker moved against the fenders there was an easy relative movement of the fenders with the tanker. The fenders easily aligned themselves with the tanker when the tanker was out of parallel and spread the impact well along the side of the tanker. When the tanker moved longitudinally they gave in a longitudinal direction fairly easily, and that reduced the wear and tear. Professor Baker did not know whether or not the Author would call the fenders used soft; they were probably intermediate between hard fenders and those to which he referred as soft.

Mr B. F. Saurin (Senior Civil Engineer, British Petroleum Co. Ltd) said that the Author used "waves" as a general scientific term, but the movements of the water in question were really what would normally be called swell. They were not local waves; in fact in the particular case under review they had been quite invisible, being entirely obscured by local sea action, which at times was several feet in height but in itself had no effect on the vessels. It was the long invisible swell which came in from the ocean which caused the disturbance. Would the Author say whether or not the Paper was confined to very exposed sites associated with large bodies of water, where very long waves could persist, even though they might be very much attenuated in height?

99. Mr Saurin's second point concerned the impacts, which Professor Baker had said were apparently so great that he could not imagine a ship withstanding them. In fact the impact of 800 tons referred to in the Paper had been measured on the model and was associated with a wave height of 2 ft, which the Author had explained had been an overestimate. That had been the first estimate of the probable height of the swell, but, as the Author had said, something under 1 ft would be nearer the mark, and the experiments which had been carried out on the model with that lower wave height had not suggested impacts of anything like 800 tons. That had been confirmed by the site report, which stated that the fenders had never been brought entirely home.

100. However, there had been damage to the plates of ships, but that had arisen from a secondary cause. The movement of the ship was not entirely horizontal but had a vertical component. The ships moved towards the jetty and when they struck the fenders they dropped. That dropping caused wear on the wooden facing timbers which were torn away gradually. After some time various fastenings such as bolts attaching the fender timbers became exposed, and those projections had caused damage to ships' plates by a very localized impact.

101. With regard to the application of the results of the research, at the site the winter which had given rise to the trouble must have been exceptional, or at least it was to be hoped so, because for the last two or three seasons since then there had been no serious movement, rather to the Author's disappointment. The results of the investigation had

been applied in principle. The original fenders had not been replaced by longer stroke fenders, but their original travel of 14 in. had been increased to about 24 in. by putting more rubber on the front of the timbers, and it was hoped that, if the winter which had given rise to the trouble had in fact been exceptional, that would be sufficient. Certainly at the moment the position was satisfactory.

102. For the design of fenders the Author had emphasized that it was the travel which was the principal characteristic. Gravity fenders were used because that was a convenient way of getting long travel, but with rubber blocks in series it had been proved possible in later designs to get travels similar to those associated with a gravity fender.

Mr N. H. Searle (Head of Hydraulics Division, Central Laboratory, George Wimpey & Co. Ltd) said that his firm had recently carried out marine borings off the south coast of England and it had been of interest to note the similarity between the behaviour of the boring vessel at its moorings and the model-ship movements described in the Paper.

104. The boring vessel had been 187 ft long and Fig. 12 showed the method adopted to anchor the ship over a borehole position. Four moorings were used, each consisting of an anchor connected by heavy chain cable to a 6-ft-dia. spherical mooring buoy. Steel wire ropes connected these buoys to winches on the ship. In the off-shore deep-water borehole positions the mooring arrangement was as shown in the section, Fig. 12. Swell waves approaching the ship beam-on had caused the vessel to surge and there had been a tendency for the mooring buoy to be dragged under water. The natural buoyancy of the buoy resulted in a relatively gentle restoring force being applied to the ship with the result that transverse ship movements were limited to 2–3 ft. The horizontal movement of the vessel was measured by reference to a borehole casing which had been sunk into the sea-bed and stood unsupported in the slot of the boring platform.

105. It had been found that ship movements were least when the steel mooring wires were not too taut and the buoys were bobbing fairly freely on the water surface. If the wires were made bar taut so that the mooring buoys were dragged under water, there was less spring in the anchor assemblies and larger horizontal ship movements occurred. There seemed to be an optimum value for the tension in the mooring wires in order to keep the ship most stable. From observations aboard ship, Mr Searle was not convinced that an unrestrained vessel would move less than a moored vessel, since when the wires had been allowed to go slack, slightly greater horizontal movements had been experienced than if the wires were fairly taut. Perhaps wind or currents had affected the vessel. Would the Author comment on the effect of these additional variables when coupled with wave forces, since obviously they had to be considered when designing new berths.

106. The lower section in Fig. 12 showed that when the ship had to be moored close inshore, the starboard anchors required to be placed on the foreshore, with the result that the anchor chain was pulled taut and the beneficial effect of the buoyancy of the mooring buoy was lost. This arrangement was similar to the stiff fender and soft mooring ropes described in the Paper. Transverse ship movements had been between 6 and 8 ft and this had made boring work very difficult. The smaller movements of the ship in the off-shore positions had been attributed to the more gentle spring action of the moorings in deep water. These experiences seemed to confirm the Author's statement that soft fenders and soft moorings would provide the best means of minimizing ship movement. Mr Searle showed a slide which included two views taken at a time interval of 7 sec. showing the way in which mooring wires had gone alternately slack and taut in phase with the wave period.

107. The problem of deciding upon a tolerable wave height and discussed by the Author in § 5 was similar to that which had faced Mr Searle's firm recently in the design of a harbour which they were building in Peru. A model had been constructed to indicate the most satisfactory length of breakwater and alignment of jetties. To help in the interpretation of this model, a small ship had been moored alongside the model berths. He then showed another slide of an operator measuring the movement of the model ship which was being held in position by thin latex yarn, so that for practical

FIG. 12.—MOORING ARRANGEMENT FOR OFF-SHORE BORINGS

purposes it represented an unrestrained ship lying at the berth. The model ship had been used at the suggestion of the Author who had advised on the construction and operation of this hydraulic model. The size of the model ship had been chosen after consideration of the tonnage of the average vessel expected to use the harbour.

108. Diagrams had been drawn to represent wave heights, total ship movements, and transverse ship movements at various points within the harbour. So far as the breakwater length was concerned the same conclusions had been reached whether wave heights or total ship movements were considered. The use of the model ship had been of value, however, since it had indicated the extent of the transverse movement and where this had been excessive, the berths had been re-sited. This information could not have been obtained from a study of wave heights or even by visual examination of the model, since reflexion and counter-reflexion of waves within the harbour obscured the pattern.

Mr Roderick Agnew (Hydraulics Lecturer, Imperial College of Science and Technology) presented a mathematical analysis for a ship moored at a jetty by several ropes, only one of which was shown in Fig. 13. Long-crested waves of wavelength λ approached the jetty at an angle of incidence α . The x -axis parallel to the jetty, the y -axis normal to the jetty, and the s -axis (in the instantaneous direction of the ship's movement in the horizontal plane) were all taken to have their origin at the ship's centre of gravity CG, which was assumed to coincide (in plan) with the centre of buoyancy and the centre of drag resistance,

FIG. 13.—MOORED SHIP STRIKING A FENDER A GLANCING BLOW

110. Symbols used in the analysis were:—

A_p	projected area of underbody in direction s .
A, B	arbitrary constants defined in the text.
B_f	distance from face of fender to ship's centre-line.
a, b, c, d, e, f	constants defined in the text.
$C_0, C_1, C_2 \dots$	coefficients.
C_d	drag coefficient.
D	hydrodynamic drag.
F	fender reaction.
H	wave-height, crest to trough.
k	$2\pi/\lambda$.
L, l	distances defined in Fig. 13.
M	inertia (mass) of ship.
m	virtual inertia (mass) of moving water.
R	radius of gyration.
r_1, r_2	roots of an equation, defined in the text.
S, s	see X, Y, S .
T	tension in mooring rope (s).
t	see X, Y, S .
W	ship's weight component down wave slope.
w	distance along wave ray (in direction opposite from W).
X, Y, S	distances moved by ship (subscript s) or water (subscript w) along x -, y -, s -axes in time t .
α	angle of incidence of waves.
β	angle between mooring rope and ship's centre-line.
γ	angle between wave ray and s -axis.
δ	wave slope.
ζ	elevation of water surface above mean.
θ	angle between ship's centre-line and x -axis.
λ	wavelength.
μ	coefficient of friction between ship and fender.
ρ	density of water.
ϕ, ψ	functions of x, y .
ω	constant of simple harmonic motion.

111. The dynamic equations were:

$$\text{x-axis: } (M+m) \frac{d^2(X_s - X_w)}{dt^2} = D \cdot \sin(\alpha + \gamma) - W \cdot \sin \alpha + \sum T \cdot \cos(\beta - \theta) + \mu F \quad (1)$$

$$\text{y-axis: } (M+m) \frac{d^2(Y_s - Y_w)}{dt^2} = D \cdot \cos(\alpha + \gamma) - W \cdot \cos \alpha + \sum T \cdot \sin(\beta - \theta) - F \quad (2)$$

Rotation about CG:

$$(M+m)R^2 \frac{d^2\theta}{dt^2} = \sum T \cdot l \sin \beta + \mu F (B_f \cos \theta + L \sin \theta) - F \cdot L \cos \theta \quad (3)$$

To get an analytical solution the right-hand side of each equation had to be expressed in terms of X , Y , and θ and linearized. The Author had dealt adequately with the virtual mass m .

112. The drag $D = C_d \rho A_p \left\{ \frac{d(S_w - S_s)^2}{dt} \right\}^2$. The index 2 of the relative velocity would make the equations non-linear, so the degree was reduced by Lorentz's linear approximation ("if the motion is approximately simple harmonic given by $V = V_0 \cos \omega t$, then the term $V^2 = V_0^2 \cos^2 \omega t \approx \frac{8}{3\pi} V_0^2 \cos \omega t = \frac{8}{3\pi} V_0 V$ "). Fig. 14 showed that the approximation consisted in taking the first term of the Fourier series for $\cos^2 \omega t$. But $X = S \sin(\alpha + \gamma)$ and $Y = S \cos(\alpha + \gamma)$, hence the x-component of the drag was:

$$D \sin(\alpha + \gamma) = \frac{8}{3\pi} C_d \rho A_p V_0 \frac{d(X_w - X_s)}{dt}$$

and the y-component of the drag was similarly:

$$D \cos(\alpha + \gamma) = \frac{8}{3\pi} C_d \rho A_p V_0 \frac{d(Y_w - Y_s)}{dt}$$

FIG. 14.—LORENTZ METHOD OF LINEARIZING SIMPLE HARMONIC MOTION

113. The ship's weight was Mg , with component $W = Mg \sin \delta \approx Mg \delta$ (the wave-slope δ being periodic). Then, assuming simple harmonic motion, $\zeta = \frac{1}{2}H \sin(\omega t + kw)$. Hence the slope in the w -direction was:

$$\frac{d\zeta}{dw} = k \cdot \frac{1}{2}H \cos(\omega t + kw) \text{ and } W = Mgk \cdot \frac{1}{2}H \cos(\omega t + kw)$$

Hence the x- and y-components of W were:

$$W \sin \alpha = Mg \frac{\pi H}{\lambda} \sin \alpha \cdot \cos \left(\omega t + \frac{2\pi \sin \alpha}{\lambda} x \right)$$

and

$$W \cos \alpha = Mg \frac{\pi H}{\lambda} \cos \alpha \cdot \cos \left(\omega t + \frac{2\pi \cos \alpha}{\lambda} y \right)$$

The mooring tension T was a function of the ship's movements. Wilson⁴ had adduced evidence that $T \propto S_s^n$ where $n > 1$, giving another non-linear relation. This could be linearized as in Fig. 15, by taking a false zero. Measuring tension in tons (of 2,000 lb.) and lengths in feet, Wilson had suggested the empirical equations for x - and y -components:

$$T_x = X_s^{6.37}/324 \text{ and } T_y = Y_s^{6.72}/285,000$$

The curves could be approximated by the straight lines $T_x = a(X_s - b)$ and $T_y = c(Y_s - d)$, where $a = 8$ tons/ft, $b = 2.5$ ft, $c = 3$ tons/ft, and $d = 6.5$ ft. Then:

$$T \cos(\beta - \theta) = a(X_s - b) \text{ for } X_s > b \text{ or } T = 0 \text{ for } X_s < b$$

and

$$T \sin(\beta - \theta) = c(Y_s - d) \text{ for } Y_s > d \text{ or } T = 0 \text{ for } Y_s < d$$

114. The fender reaction F could similarly be approximated by a straight line $F = e(Y_s + f)$, where the constants e, f , obtained from Fig. 3, were:

$e \approx 280$ tons/ft and $f = 0$ for existing fenders.

$e \approx 7$ tons/ft and $f = 1$ ft for proposed gravity fenders if deflexion $Y_s >$ about 3 ft.

$F = 0$ if the ship is not in contact with fenders.

The straight line represents $T_x = a(X_s - b)$.

The curved full line refers to experimental values of T_x .

The curved broken line is Wilson's curve.

FIG. 15.—APPROXIMATION BY FALSE ZERO

115. The rubbing friction μF , assuming no deflexion of the fenders in the x -direction, could be determined by assuming a suitable friction coefficient μ .

116. Then, Mr Agnew assumed a periodic horizontal water movement with components $X_w = X_0 \cos(\omega t - \phi)$ and $Y_w = Y_0 \cos(\omega t - \psi)$. Hence:

$$\frac{dX_w}{dt} = -\omega X_0 \sin(\omega t - \phi) \text{ and } \frac{d^2 X_w}{dt^2} = -\omega^2 X_0 \cos(\omega t - \phi)$$

and similarly for derivatives of Y_w .

117. Substitution in equations (1) and (2) gave the following pair of linear second-order differential equations, with coefficients which could be determined from the preceding detailed analysis:

$$\frac{d^2 X_s}{dt^2} + C_1 \frac{dX_s}{dt} + C_2(X_s - b) - \mu C_0(Y_s + f) = C_3 \cos(\omega t + K) \quad . \quad . \quad (4)$$

$$\frac{d^2 Y_s}{dt^2} + C_4 \frac{dY_s}{dt} + C_5(Y_s - d) + C_0(Y_s + f) = C_6 \cos(\omega t + \epsilon) \quad . \quad . \quad (5)$$

The solution of equation (2) could be written in the standard form:

$$Y_s = \frac{C_5 d - C_0 f}{C_5 + C_0} + A e^{r_1 t} + B e^{r_2 t} + C \cos(\omega t - \phi) \quad . \quad . \quad (6)$$

where the arbitrary constants A and B were equal to zero after transient stage and r_1 and r_2 were the roots of the characteristic equation:

$$r^2 + C_4 r + (C_5 + C_0) = 0, \quad C = \frac{C_6}{\sqrt{\omega^2 C_4^2 + (C_5 + C_0 - \omega^2)^2}} \quad \text{and} \quad \tan(\phi + \epsilon) = \frac{\omega C_4}{C_5 + C_0 - \omega^2}$$

Equation (6) could then be substituted in equation (4) to give the solution for X_s , which would be another case of damped harmonic motion.

118. Finally, mooring tension T was given by $T = \sqrt{T_x^2 + T_y^2}$ where $T_x = a(X_s - b)$ and $T_y = c(Y_s - d)$.

Mr C. Cuthbert (Rubber Technologist, The Leyland & Birmingham Rubber Co. Ltd, Preston) said that although the Author had used gravity fenders to demonstrate the effects of very soft fendering, the work had really shown the difference between two basic types of fendering: one which might be called medium hard, with 14-in. travel, and the other a very soft fender with long travel. The comparison in the experimental work was really between those two types and not necessarily between rubber and gravity fenders.

120. There was no great difficulty in devising a rubber fender with long travel. There were certain restrictions, but it was now possible and developments in design were now going on to increase the travel with rubber fenders. Mr Cuthbert had recently done some experimental work with very soft rubber with which it was possible to get 3-4 ft travel with 100 tons thrust.

121. In developing the design of very soft fenders, had the Author considered the total energy to be absorbed during berthing? There seemed to be two distinct requirements from fenders: first, the actual berthing, when the ship was not restrained by a rope, and secondly, the behaviour of the ship after berthing, when roped up. The total energy capacity of the prototype gravity fenders appeared to be a little on the low side. It had been interesting to learn from Mr Saurin that the 800-ton thrust mentioned in the Paper had been arrived at as a result of using a wave-height which possibly was not an exact replica of the waves encountered in practice, because 800 tons on the rubber fenders, pushing them right back, would be equivalent to the full energy of a 32,000-ton tanker moving at 1 ft/sec. If under certain conditions such a high energy had to be absorbed, the ideal fender would be a combination of a very soft fender initially with sufficient ultimate capacity to absorb the maximum energy likely to be encountered. By combining the various hardnesses of rubber in series, it was possible to change the load/compression curve to give a very soft rate initially and a stiffer rate finally, and so obtain a soft fender to meet the Author's experimental requirements yet with adequate total energy absorption.

Mr E. I. Loewy (Senior Engineer, Sir William Halcrow & Partners, Consulting Engineers) asked if time had prevented the use of models before the whole project had been designed.

123. Professor Baker had noted the absence of reference in the Paper to ship distortion. Could the Author state the possible influence of that in this problem? It was well known that not very long ago most wharfs, and even now very many, had no significant energy-absorbing facings of any kind, and yet thousands of successful berthings were going on and had been going in for many years. Ship distortion must surely have been the operative factor in these cases.

124. Since the 1939-45 war a great deal of work had been done on the development of resilient fendering systems, prompted very largely by the great increase in the size and number of tanker vessels. Although no finality had been reached, many useful ideas were being developed, and the challenge of these very large vessels from the point of view of fendering, was being met with reasonable success. However, there had been no comparable development on the mooring of such large vessels. All that was being offered was more and bigger and better ropes.

125. There must be some totally different method of mooring large vessels which ought to be thought about, such as securing vessels magnetically in some way or making

use of the analogy of mooring airships at one end, or possibly at both ends, by some contrivance, the aim being to secure the vessel quite rigidly. Could the Author suggest what would be the forces that any completely rigid structure would have to withstand, because it seemed that it was the mobility of the vessel which had such a big influence on mooring forces.

126. Some of the examples which the Author had quoted from his model seemed to be rather incomplete, because the relation between the slope and period of the wave must have some effect, since the vessel had to be accelerated significantly before the wave had reversed its slope. The forces concerned were related to the period of the wave as well as to the slope. The 20-sec waves which the Author described were for British conditions very long, and 7-8 sec would be more usual. The circumstances would then be very different.

Mr Alfred Hiley (Consulting Engineer) observed that Fig. 5 showed the principle of the gravity fender. Such fenders, each weighing about 15 tons, were able to yield backwards with a force which would gently take charge of the ship when she approached and surged against the wharf. Fig. 6 showed the advantage of this type of fender, as compared with the existing ordinary type of stiff fender, in reducing the force of resistance to impact on the ship from a very considerable value—up to 800 tons—down to 1/10th or 1/20th, which was a very important achievement. Fig. 4 demonstrated the action of the fender very clearly. In resisting the inward motion of the fender some form of leverage could possibly be used to give an initial outward force, which would give a more constant resistance to the forces required to move the fender.

128. The advantages of the fenders shown in Fig. 4 were recognized, but this other method, § 74 (f), could supplement them effectively if holding-off buoys could be made to take an outward pull from seawards to the ship at the height of the upper deck, at which level she is held fixedly against the initially opposing pull as set up in landward hawsers, so that she would not harm the fenders or even send them inwards more than a predetermined convenient distance lower down.

129. In connexion with holding-off buoys, with anchorage ropes, presumably the Author had in mind the application of mooring anchors and weighty concrete clumps, some of which might even weigh up to 10 tons each; but there was a limit to what could be expected from anchors of this type. Thorpe had made a careful analysis⁵ of the energies that could be taken up by raising the chains to greater tautness, and the resistance of an anchor to dragging, which was generally no more than about 6 times its own weight. With, suppose, an 8-ton anchor, such as employed by big tankers and aircraft carriers, that would mean, if some degree of safety be preserved, a force of about 50 tons per anchor, which did not, however, amount to very much compared to the down-wind force of anything up to 100 tons which might have to be resisted on the side of the vessel in some extreme cases.

130. What general depth of water, if more than 6 or 8 fathoms, did the Author contemplate dealing with in putting anchors down for mooring purposes for a hold-off buoy? The problem could then presumably be tackled from another standpoint in that it would enable vertical soft fenders, whether rubber or other types shown by the Author's model as 4-5 ft dia., to be utilized, to have a comparatively large yield, and be gentle in their action, but the ship would be held off from the structure of the wharf by, say, 4-5 ft and be kept there by the holding-off mooring ropes, which must be effectively anchored. What anchorage-resistance should thereupon be provided? If it was less than 50 tons before commencing to drag it would not be enough, and even 50 tons in the case of big vessels often might be inadequate. It might be useful, therefore, for the Author to extend his investigations to see if it would be possible for the civil engineer to provide anchorages in the ground or in a sandy sea-bed for securing the mooring cables from the buoy and which would resist without movement a force of, say, 200 tons. **Mr Hiley** felt sure it could be done.

131. The friction on a sandy bed was not, as a rule, more than half the weight of any cable laid upon it, so that in the present context it was insignificant, but an anchorage,

buried about 30 ft deep, or "hold-fast" pile cores in sets of five driven from a floating Diesel plant, and scientifically deposited, could provide against an uplift of perhaps 300 tons to equal the strength of the actual chain cable in use, and arrangements could be made for a given margin of yield by the soft fenders before the ship could come in contact with the fenders on the wharf along the waterline.

The following contributions were received in writing.

Dr Basil W. Wilson (Professor of Engineering Oceanography, Agricultural and Mechanical College of Texas, College Station, Texas, U.S.A.) observed that studies of ship motion under restraints from mooring lines and fenders attached to some fixed structure had tended hitherto to devote principal attention to the longitudinal (surge) motion of the ship excited by the action of progressive or standing waves. While mooring-line failures were usually attributable to the longitudinal surging of a ship, surging of itself was not necessarily dangerous or damaging unless a ship, broken adrift, was in imminence of collision with nearby ships or structures. Lateral sway motion, on the other hand, was always potentially dangerous, since it imperilled both the ship and the mooring structure alike and, in cases of severe uncushioned impact, could lead to crushing and buckling of their structural components. Relatively, therefore, the problem of sway motion was more important.

133. In justifying his model approach the Author had commented upon the complexity of actual ship motions and the seemingly anomalous behaviour of moored ships under the influence of an irregular sea. Somewhat similar experience in 1945 had led Dr Wilson to theoretical and model studies⁴ of the surge and sway problem, though with rather limited consideration of the idealized sway motions of a ship. Undoubtedly these means afforded the only way of acquiring understanding of the complex composite motions that obscured ship behaviour.

134. In § 7 the Author had concluded that 20-sec-period storm waves of low height were probably responsible for the movements of the moored tankers at the study site. Such a conclusion would not be unreasonable in the light of findings elsewhere^{4, 6, 7}, but it could also be that other periodicities in the swell might have been of importance. While Dr Wilson would not expect that the results of the experiments, in so far as general principles were concerned, would be too much affected by restriction to only one wave period, nevertheless, it would seem desirable that the experiments had been conducted over a range of wave frequencies, particularly in the second series of experiments.

135. In the Author's statement of the equation of sway motion of the ship (§ 36) Dr Wilson would prefer to see the negative sign removed from F_m on the grounds that F_m was a restoring force and not a driving force. Also the value of d^2X_w/dt^2 was really $[-g \times (\text{slope of water surface})]$ making the equation of § 38:

$$F_m = -(M+m)[g(\text{slope}) + d^2X_s/dt^2]$$

in which positive slope rose, and negative slope fell, in the direction of positive X_s .

136. Dr Wilson would define $(M+m)$ as the virtual mass or apparent mass of the ship, and m as the added mass resulting from the hydrodynamic pressures of the water on the ship, and believed that this terminology had more common usage than the definition of m as virtual mass. The Author had rather tacitly assumed that the ratio (m/M) for a floating semicylinder of the underwater shape of a ship was likely to have a value between 0.5 and 1. This assumption could be gravely in error, depending on the equivalent block dimensions of the ship and the direction of its horizontal motion. Thus for a (block-) length-beam ratio of a ship of from 5 to 6, as was common, and for transverse motion, m/M tended to values between 2.35 and 2.82 (cf. Fig. 7 of reference 8). These values were validated both theoretically and experimentally.

137. In using a model 24.8 in. long $\times$ 17.4 in. wide, the length-beam ratio would have been 1.43 and the m/M ratio 0.81, as compared with the prototype full-size ship (block-) length-beam ratio of 4.22 and m/M ratio of 2.05. Thus in terms of the equation

⁴ References 4 *et seq.*, are given on p. 450.

of § 38, the model, even corrected to the prototype mass, would underestimate the restoring forces in the ratio $(1+0.81)/(1+2.05)$ or about 0.6. Model interpretations of force might thus be expected to be as much as 40% too low on this score alone.

138. Dr Wilson believed, however, that the Author's experimental results, while qualitatively demonstrative of the nature of the sway motion, should be treated with considerable reserve with regard to adaptation to prototype conditions, and for other reasons.

139. First was the fact (on the Author's own admission) that the spring characteristics of the model "mooring rope" and "fender" springs did not have the same non-linear characteristics as the prototypes. The model spring rates were stated to have been similar to those of the ropes with the spring characteristics given in Fig. 11. But the applicability of some of the curves of Fig. 11 was open to question for the reason that they had no resemblance to the true load-sway-movement relation that actually prevailed in the direction transverse to the ship. In illustration of this point Dr Wilson presented Fig. 16, based on his previous work⁴. Fig. 16a showed the load-extension relation for a typical mooring line comprising a 71-ft-long $3\frac{1}{2}$ -in. steel wire attached to an 18-ft-long 18-in. coir sling; Fig. 16b, on the other hand, gave the horizontal-component

FIG. 16.—(a) LOAD/EXTENSION RELATION FOR A TYPICAL COMPOSITE MOORING LINE; (b) CORRESPONDING RELATION BETWEEN TRANSVERSE HORIZONTAL RESTORING FORCE AND LATERAL SWAY MOVEMENT OF SHIP.

tension at the ship, transverse to the ship, developing in the same line as result of lateral movement of the ship in a typical mooring situation. The differences resulted from mooring-line geometry and the obliquity of the mooring line to the transverse direction. The two curves were obviously dissimilar, that of Fig. 16b being strongly non-linear. To have approached this type of relation, the cantilever "mooring-rope" spring of the model would have had to have a curved bearing such that its cantilever length diminished continuously with increase of end deflexion.

140. A second reason for the need for exercising care in the interpretation of the

model results on a prototype scale was that not all the curves of Fig. 11 were held to be valid under operating conditions. The result for the 4-in. rope, for instance, reproduced from reference 4, was strictly a median relation taken from the test repeated-loading hysteresis-loop. This was shown again in Fig. 17a, which gave also equivalent test results for 18-in. coir and 8-in. nylon rope. Dr Wilson's data for the 8-in. ("Viking") nylon rope (obtained originally from British Ropes Ltd, Edinburgh) agreed closely with that of the Author (Fig. 11) in respect of the initial load-extension relation, but the repeated-loading curves, after permanent and semi-permanent set, were vastly different

FIG. 17.—ELASTIC CHARACTERISTICS OF TYPICAL MOORING ROPES. (a) INITIAL LOADING BEHAVIOUR; (b) APPROXIMATE REPEATED LOADING BEHAVIOUR AFTER TEMPORARY SET.

from either. Yet it was always the repeated-loading relation that applied under practical conditions where a ship was subject to more or less continuous sway motions (usually coupled with surge and yaw). Thus, the applicable relation should tend to follow Fig. 17b rather than Fig. 11.

141. On the "fender" spring side of the model, it seemed hardly likely, too, that a simple cantilever spring would stand much chance of reproducing the extreme non-linearity of the prototype gravity fender, of the design shown in Fig. 5, and this fact also somewhat endangered the conclusions. In any case Dr Wilson was not convinced that

the gravity fenders, with the properties shown in Fig. 3, were necessarily the best type of cushioning for the control of sway motion. The non-linearity of the force-deflexion relation seemed too extreme, with the result that end-loads at full deflexion could run dangerously high. The same amount of energy could be absorbed by fenders that were less soft, with end-loads that were much less. This was exemplified in Fig. 18 in which a hypothetical fender system with the same energy-absorption capacity as the gravity fenders was shown with an end-loading of only 165 tons and deflexion of 3.37 ft, compared with the end-loading of 400 tons and deflexion of 5 ft of the gravity fenders. It would thus seem that some optimum design existed between curves A and B of Fig. 18.

142. Dr Wilson believed that the Author had otherwise made a commendable attempt to broach the mooring problem and hopes that it will be pursued further towards the yet more intricate features of coupled motions. He also strongly upheld

FIG. 18.—ELASTIC CHARACTERISTICS OF THE FENDERS

the Author in respect of the urgent need for research to determine the threshold amplitudes for long waves in harbours, below which ship perturbations could be more economically controlled by elastic fender systems than by structural changes to the harbour itself.

Mr A. H. Beckett (Partner, Sir Bruce White, Wolfe Barry & Partners, Consulting Engineers) observed that the problem was different from the design of quays for berthing impact, and damage could not be dismissed as the result of an accident to be paid for out of insurance. There was, however, no doubt that the solutions to the problem of wave impact would also satisfy a requirement for rough berthing without damage.

144. As the demand for tanker berths increased, so were the sites for them chosen in more and more exposed areas and it was inevitable that the problems described in the Paper would have to be faced sooner or later. The very interesting results given in Fig. 6 make an overwhelming case in favour of long-travel gravity fenders. The Author, in his conclusions, had said that the satisfactory use of new jetties in exposed sites would depend on the use of soft fenders with unprecedented long travel.

145. That was the experience drawn from model tests but it was completely supported by the experience of Mr Beckett's firm with their large long-travel gravity fenders. They had been built weighing up to 600 tons in ring form and having multiple above-water suspension giving more than 3 ft of travel. Unnecessary movement of the fenders was restrained by rubber-bushed linkages so that the fenders were not noisy with constant movement and wear. The fenders had been introduced about 5 years previously to deal with berths exposed to seasonal waves and swell and during 5 years of constant use there had been no significant damage to ship or structure.

146. One could believe that the investigation on the Kwinana jetty by the Department of Scientific and Industrial Research, being the first of its type, might well have been expensive. The techniques now developed, however, might be of great use to engineers with similar problems. Could the Author give some idea of the likely cost of a similar study?

The Author, in reply, said that the influence of the weight of a gravity fender (§ 95) had not been investigated, but it could sometimes be predicted. Under the conditions that the Author had recommended for the moorings, i.e., where the stiffness of the fenders and the stiffness of the rope system was so small that the ship oscillated as if unrestrained, doubling or halving the weight of gravity fenders would have negligible influence on the movements of the tanker. The forces between the fender and the ship would simply be doubled or halved. That was the situation when the fender stiffness was very much less than the critical value, which resulted in a natural frequency equal to the wave period (§ 60). For example, the mean stiffness of the 15·1-ton gravity fender taken to a deflexion of 2 ft was found to be 14 tons/ft, and that would be described as soft.

148. If the stiffness of the fenders was as much as 60 tons/ft, as in Fig. 9c, doubling their weight would increase the movements of the ship (*cf* 9b and 9d); the force between the fender and the ship would then be more than doubled.

149. To allow the ship to oscillate farther from the quay and possibly to reduce the impacts, the mooring ropes had been slacked off to different extents in the experiments on the oil tanker. The results given in Fig. 6 showed that with the existing fenders this brought about no consistent change in the impacts. When, however, the tanker was loaded and moored against gravity fenders (which was the case when the tanker's motion was nearest to that of an unrestrained ship) the looser ropes resulted in lower impacts. The amplitude of the oscillations appeared to be unaffected. It remained that of an unrestrained ship, but the ship took up a mean position farther from the jetty.

150. To Mr Saurin the Author replied that the experiments were concerned with waves of only one length, the length appropriate to a period of 20 sec. The findings of the Paper would not be applicable to the case of a ship moved by the very long waves that had periods in the region of 200 sec and heights of less than 1 ft, nor to waves that were short compared with the beam of the ship. In the case of the very long waves of low height, the forces required to hold the ship stationary, being the product of the slope of the water surface and the sum of the mass of the ship and its hydrodynamic mass, were quite small. The forces could be provided by tight stiff ropes which then constituted the best type of mooring. Moreover, the horizontal free motion of the ship in those very long waves might be so great that it was no answer to leave the motion of the ship unrestrained. Waves that were short compared with the ship's beam had little influence on it.

151. Mr Searle's contribution to the discussion showed the practical limitations to the Author's contention that the softest moorings were the best. Obviously, the softer the moorings were, the more the ship would move under the influence of unsteady winds and unsteady tidal currents. In such circumstances the Author would, as a first trial, select a mooring system in which the natural period of the ship's oscillation on its moorings would be about three times the period of the waves. The theory of damped linear vibration suggested that little reduction in amplitude of oscillation due to wave motions would be achieved by any further softening of the moorings beyond this point.

152. The general three-dimensional problem, in which there was slack in the ropes and the elasticity of the ropes was different from that of the fenders, would never be solved analytically, and he was not surprised that Mr Agnew was unable to solve the equations that he had set down. He did not see how the method of moving the zero of the load/deflexion curve twice every wave period could be handled. A solution had to be found for complete oscillations, each being made up of a period when the ship was in contact with the fenders, two periods when the ship was not restrained mechanically, and a period when the ship was restrained by the mooring ropes.

153. It was indeed possible to hold a small ship stationary in waves (§ 125), using a reasonable number of ropes of reasonable size, but it became increasingly difficult as the size of the ship increased. To keep a ship still, it was necessary to provide a mooring so stiff that the natural period of the ship's oscillation was less than one-half, say, of the wave period. In the case of a 32,000-ton-deadweight tanker moored in waves of 20-sec period the necessary stiffness would be $\frac{4\pi^2 \times \text{mass}}{(\frac{1}{2} \times \text{wave period})^2}$ (neglecting the hydrodynamic mass) or 1,870 tons/ft. Such a mooring would require about 100 times as many wire ropes as were used in practice.

154. The Author had no information about distortion of ships. He had thought that compared with the stiffness of the fenders under examination the ships would be very stiff, but he allowed that this might not be so.

155. In view of the doubts that Dr Wilson had raised about the meaning of virtual mass, the Author had looked up a variety of references and found both the possible definitions. Hereafter he would refer to the hydrodynamic mass when indicating the mass of water that appeared to be associated with the ship.

156. Dr Wilson was correct in pointing out that an error was involved in assuming that the hydrodynamic mass of the ship was 1 and that that of the section of the ship was 0.5. Since the publication of the Paper, some experiments had been made at the Hydraulics Research Station on the hydrodynamic mass of ships. They were in connexion with a further study of two-dimensional symmetrical motion which it was hoped to describe in a Paper to the Coastal Engineering Conference in 1960. It had been found that the hydrodynamic mass was indeed very different from 1, particularly when there was only a small clearance between the bottom of a ship and the sea-bed. The results were similar to some published by Dr Grim⁹ in a Paper that had just come to hand. It was found possible to obtain a value for the hydrodynamic mass as high as 4 and certainly, by choosing an appropriate depth of water, Dr Wilson's figure of 2.35-2.82 could be obtained. Experiments on the ship's section had revealed that its hydrodynamic mass was 0.8, almost exactly the figure predicted by Dr Wilson, using the method of Browne, Moullin, and Perkins.

157. The Author was grateful to Dr Wilson for referring him to that Paper, although he had found it not really applicable to a short rectangular body, but only to very long bodies oscillating along a path normal to their length, in which flow round the ends was unimportant compared with flow underneath the body.

158. The results obtained by scaling-up data obtained from the ship-section model might well be in error by 40%. This was rather a small error compared with the 20-to-1 range of forces and 10-to-1 range of total travels that were possible when the same ship was moored in alternative waves. The error, now that Dr Wilson had pointed it out, could be allowed for.

159. Dr Wilson had complained that the characteristic curves of the moorings used in the model were different from his typical one given in Fig. 16b. The curves were markedly different when there was no slack travel, but in half of the experiments referred to in Figs 9 and 10 there had been 2 ft of slack travel and then the form of Fig. 16b was almost precisely reproduced.

160. It was interesting to note that the effect of slackening the ropes by 2 ft had only a small influence on the forces in the fenders and ropes, and that it was a rather random direction.

161. The Author had intentionally not reproduced in the ship-section model the

gravity fenders of the full model. In the full model gravity fenders were used because they were thought to have the best load/deflexion characteristic for minimizing impact forces; in the ship-section experiment, which was basic, a fender with a substantially linear characteristic was preferred, because then only one quantity was required to define its stiffness.

162. Dr Wilson would surely, on reflection, like to withdraw his contention that the hypothetical fenders (c) were better than the gravity fenders (b), whose load-compression curves were shown in Fig. 18. Indeed, (c) would be better than (b) if the problem were one of how to bring to rest a ship having known energy in such a manner that the maximum force achieved was minimum. This, however, was not the problem, and the energies stored in different fenders at the limit of an oscillation due to wave motion were unlikely to be the same. Energy was not a helpful concept in this case.

163. If the stiffness of the fenders was small, as the Author was recommending, the oscillation of the ship was substantially unaffected by the stiffness of the fenders, and the minimum fender-force was achieved by having the softest fender. The Author re-affirmed that a rising characteristic such as applied to gravity fenders was the best for this application because for any given energy absorption that was required when ships berthed, the stiffness of the lower part of the characteristic curve was small.

164. The Author assured Mr Beckett that the cost of the investigation was a small fraction of the cost of the fenders. The actual costs were confidential, but the Hydraulics Research Station would always make an estimate of the cost of a similar investigation for a potential client. The costs of the first and subsequent investigations would not be very different.

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