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Further tests on braced frameworks †

by

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Discussion

Mr J. G. Nutt (Department of Civil Engineering, University of Manchester) observed that the Author had conducted an admirable series of tests on triangular frames of rolled steel sections in an attempt to show that the theory advanced will apply also to these and not only to rectangular heat-treated sections. It was a pity that the initial imperfections of these frames could not be measured, for, by deriving a value for these from the experimental deflexion curve, the only fact which could be proved was that the deflexion curve was hyperbolic within the elastic range.

19. A similar series of tests had been conducted at the University of Manchester in an attempt to evaluate the relative effects of lack of straightness, end eccentricity, and secondary moments on the value of the initial imperfections. Two types of frame had been tested—triangular frames similar to the Author's and two-bay Warren trusses, all made from $1\frac{1}{2}$ -in. $\times$ 1-in. rolled steel sections which were welded to blocks at the joints. It was intended to publish a detailed account of this investigation later, but some points were worth mentioning here.

20. Only the lack of straightness of the compression member could be measured directly from the frame. The eccentricities at the end of each member could be considered to give rise to a moment if the joint was prevented from rotating. In the truss shown in Fig. 12, P_i and e_i were the axial load and end eccentricity respectively of the i^{th} member, and the eccentricities at joints A and B of member 1 were described by e_{1A} and e_{1B} .

FIG. 12

21. The total moment at A prior to release of this joint was:

$$P_1 e_{1A} + P_3 e_3 + P_4 e_4 = \sum (P_i e_i)_A \quad \dots \dots \dots (1)$$

After release, moment taken by member 1 was:

$$-\sum (P_i e_i)_A \cdot \frac{s_1 k_1}{T_A} \quad \dots \dots \dots (2)$$

where $T_A = \sum S k$ at joint A.

The moment carried over from joint A to joint B was:

$$-\sum (P_i e_i)_A \cdot \frac{s_1 k_1 c_1}{T_A} \quad \dots \dots \dots (3a)$$

† Proc. Instn civ. Engrs, vol. 10, p. 503 (Aug. 1958).

Likewise, the moment carried over from B to A was:

$$-\sum(P_i e_i)_B \cdot \frac{s_1 k_1 c_1}{T_B} \quad \dots \quad (3b)$$

where T_B = stiffness of joint B.

If joint A was now released, the total moment in member 1 became:

$$(P_1 e_1)_A - \sum(P_i e_i)_A \cdot \frac{s_1 k_1}{T_A} - \sum(P_i e_i)_B \cdot \frac{s_1 k_1 c_1}{T_B} + \sum(P_i e_i)_B \cdot \frac{s_1 k_1 c_1 \cdot s_1 k_1}{T_A \cdot T_B} \quad (4)$$

the effective eccentricity at end A of member 1 was:

$$e_{1A} - \frac{1}{P_1} \left[\sum(P_i e_i)_A \cdot \frac{s_1 k_1}{T_A} + \sum(P_i e_i)_B \cdot \frac{s_1 k_1 c_1}{T_B} \left(1 - \frac{s_1 k_1}{T_A} \right) \right] \quad (5)$$

It had been found from these series of tests, especially in the case of the Warren trusses, that if the last term of equation (5) was neglected, an accurate correlation between elastic stability line and experimental deflexion curve was not possible. Also, as the load on the frame increased, the stiffness of the compression member relative to the other members at its ends decreased, but its carryover factor c_1 increased. The effect of the eccentricities of the compression member and other members meeting at its ends was then continually changing, and the effective $a+e$ term would be dependent upon the load on the frame. It had been found, however, that over a range of different frames the $a+e$ term had stayed nearly constant as the load varied.

22. The secondary moments could be expressed as equivalent end eccentricities, but it had been observed that for frames of either the Warren-truss type or the triangular type with uniform members, their effect on initial imperfections was small.

23. Table 3 did show one important point—that the value of $a+e$ which should be used for the elastic stability line was approximately the same as the lack of straightness of the members, and if an assumed value of $a+e$, consistent with the size of the frame were used in the determination of the elastic stability line, no serious errors should result. Various formulae for the value of initial lack of straightness had been suggested in the past and it would appear likely that one such formula would give consistent results in the above method.

The Author, in reply, pointed out that it had not been his intention to prove that the deflexion curves of the A frames were hyperbolic as Mr Nutt suggested. Rather it was the reverse in that the deflexion curves had been assumed hyperbolic to obtain an estimate of the initial imperfections. The procedure described had been adopted to overcome the lack of reliable information about the eccentricities of the members at the joints. The assumption that the deflexion curves were hyperbolic was based on previous tests (see reference 1, p. 515).

25. The exact method for obtaining the equivalent eccentricity at the joints of the member whose P/P_E -value was greatest (referred to as the critical member) was to hold all of the joints and release them in turn. That, of course, required extensive computation and the framework could be simplified on the assumption that those members remote from the critical member had little effect upon its behaviour. Mr Nutt appeared to have simplified his trusses to those shown in Fig. 13. That was valid but the moment distribution which gave equation (5) as its result was incomplete. The exact moment M_{AB} could be calculated as followed:—

Suppose A and B were held by moments $M_A = (\Sigma Pe)_A$ and $M_B = (\Sigma Pe)_B$. After rotating A and B through θ_A and θ_B respectively the total moment at A was

$T_A\theta_A + s_1c_1k_1\theta_B + (\Sigma Pe)_A$ and similarly the moment at B was $T_B\theta_B + s_1c_1k_1\theta_A + (\Sigma Pe)_B$. Since A and B were now released these moments could be equated to zero and θ_A and θ_B evaluated. The total moment in member AB at A was

$$M_{AB} = s_1k_1(\theta_A + c\theta_B) \\ = \frac{s_1k_1}{(s_1c_1k_1)^2 - T_A T_B} [(\Sigma Pe)_A(T_B - s_1c_1^2k_1) + (\Sigma Pe)_B c_1(T_A - s_1k_1)]$$

26. Unfortunately, Mr Nutt had not defined what he meant by $(a+e)$. The parameter a used in § 7 was made up of three parts and it could be written:

$$a = a_0 + \frac{e_A + e_B}{2} + \alpha W$$

where a_0 denoted the initial lack of straightness of the critical member, $\frac{e_A + e_B}{2}$ the effective eccentricity of the critical member, and αW the deflexion arising from changes of axial length of the members of the framework. The method of evaluating a was given elsewhere.^{1, 3} Equation (1) was a way of magnifying these initial effects, which

FIG. 13

became very large as the critical load W_c of the framework was approached. The term a used by Mr Nutt was presumably the term A_0 used by the Author. Without knowing full details it was difficult to give a reason for the small effects of $(a+e)$ and secondary moments which Mr Nutt had noticed. As a typical example, Fig. 3 showed that if the critical load was large compared to the yield load, i.e. if the frames were stocky, the effects of initial imperfections were difficult to measure. It was suggested that perhaps the frames tested by Mr Nutt were of this type.