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Determining run-off from rainfall†

by

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Discussion

Mr L. H. Watkins congratulated the Author for pointing out, probably for the first time, that the techniques used by hydrologists to deal with natural catchments could also with some modifications be employed by engineers concerned with urban drainage. It was possible, however, that in emphasizing the similarities of the two problems he had underestimated the differences between them.

67. It was important to remember that since the urban drainage system was artificial, its drainage pattern might well be different from that of a natural watercourse draining the same area. Since the drainage pattern at an urban area was something peculiar to the particular area, it also followed that a unit hydrograph derived from recordings at one urban area could not with confidence be applied to another apparently similar area. As a compensation for this disadvantage, it was also the case that the velocities of flow in the sewer system and hence the time of concentration and the time-area diagram of an urban area could be calculated with far greater precision than was possible in the case of a natural catchment. These calculations were also made simpler with an urban area by the fact that the water remained on the ground for a much smaller proportion of time than in a natural catchment.

68. It was for these reasons that the time-area method of calculating run-off hydrographs developed at the Road Research Laboratory, and mentioned by the Author, was considered to be more suitable for application to urban areas than any known variation of the unit-hydrograph method. The Author's objections to this type of method, which were of course valid for natural catchments, were not valid for urban areas for which every parameter employed could be directly calculated from the dimensions and gradients of the sewer system.

69. The analysis of the "rational" method given by the Author was of considerable interest. From his statement in § 28 it appeared that this method assumed a linear time-area diagram. It was worth noting that at one area (not urban) at which an investigation into rainfall and run-off had been carried out by the Road Research Laboratory the time-area diagram was in fact reasonably linear, and one might suppose therefore that if this assumption were the only weakness of the method the calculated rates of run-off for this area would be in good agreement with the recorded values. The calculated values were, however, considerably greater than the recorded values and this discrepancy was considered to be due first to the assumption of "bank-full" velocities in calculating the time of concentration, and secondly, to the neglect of the effect of temporary retention of water in the channel. At urban areas, where the time-area diagrams were far from linear, the effect of this non-linearity was usually to cancel out approximately the over-estimation due to the neglect of the temporary retention of water in the sewers. The only case discovered so far of an urban area at which this compensating effect had not resulted in the "rational" method giving fairly accurate results was at a

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drainage area where the sewers at the head of the system were excessively large. As a consequence, the actual velocities of flow were smaller than they would have been if the sewers had had a normal amount of taper, and the calculated rates of flow were therefore greater than the recorded values. Investigations were continuing at other drainage areas in order to determine any other limitations that the rational method might have when applied to urban areas.

70. The assumption of "bank-full" velocities and the neglect of the retention of water in the river seemed undoubtedly to be the causes of the discrepancies between O'Kelly's observed and calculated unit hydrographs, quoted by the Author. The differences between O'Kelly's unit hydrographs derived from the time-area diagrams and from the "empirical method" were precisely the differences found at the Road Research Laboratory between hydrographs calculated from time-area diagrams and the same hydrographs after correcting for temporary retention.

Professor J. C. I. Dooge congratulated the Author on this Paper which placed the different versions of the rational method in their proper perspective as special cases of the unit-hydrograph approach. Other commendable features were the unified treatment of urban and rural run-off, and the reduction of the unit-hydrograph approach to its simple mathematical terms, so that the analogy with similar problems in other fields became evident. The adoption by the engineering profession of the viewpoint suggested by the Author would lead to real progress in applied hydrology. The following comments represented differences of emphasis and minor disagreements which did not detract from the fact that the writer was in general agreement with the Author. The comments are grouped under the same headings as used in the Paper.

72. The Author appeared to treat the relation between rainfall and run-off as a single hydrological problem divisible into three parts, i.e. determining (a) the volume of run-off, (b) the time-distribution of run-off, and (c) the relation between rainfall frequency and discharge frequency. The present writer thought it was more helpful to distinguish between several standard hydrological problems each of which contained several parts. As a basis for discussion, the following classification was suggested. There were three classical problems of flood hydrology:—

- (a) predicting the maximum flood,
- (b) predicting a flood of given frequency,
- (c) predicting an individual flood.

73. Each of the above classes of problem could be sub-divided according to the quantity and type of information required, as follows:—

- (a) determining the peak discharge of the flood,
- (b) determining the duration for which the discharge exceeds a certain value,
- (c) determining the complete hydrograph of the flood.

74. This classification distinguished nine basic problems in flood hydrology; the techniques most suitable for their solution might differ from problem to problem and techniques developed for one problem might be extended to others. Thus, the rational method was originally proposed by Mulvaney⁵ to solve problem 1a (i.e. the prediction of the peak of the maximum flood) and was later modified to cover other problems as well. Similarly, Sherman¹⁵ devised the unit hydrograph to solve problem 3c (i.e. the prediction of the hydrograph of an individual flood) and it too was applied to other problems.

75. There were two main avenues of approach to the solution of any hydrological problem. The first involved the use of statistical methods either in the crude form of universal empirical formulae or by means of the more sophisticated statistical techniques now available. The second approach was the physical one in which an attempt was made to develop a reasonable theory of flood run-off based on physical principles. The latter approach, of which the rational method was the first step, was the one adopted

by the Author, and there could be little doubt that it would prove the most fruitful in the long run.

76. In the Paper the classical methods of determining urban run-off were divided into three groups—the rational method, the tangent method, and the time-area methods. The restriction of the term time-area methods to the last group was not quite satisfactory, as the time-area diagram was also essential in the second group (tangent method). The distinction between “the tangent method” and “the time-area methods” and the sub-division of the latter group both depended essentially on the nature of the assumption in regard to the distribution of rainfall through the storm. Accordingly, it might be preferable to consider all these methods as variations of the rational method and to classify them as in Table 1.

TABLE 1

	Rainfall assumption used	Authors	Nash's terminology
A	Uniform intensity (full area)	Mulvany, ⁵ Kuichling, ¹⁷	Rational method
B	Uniform intensity (partial area)	Chamier, ³² Lloyd-Davies ³	
C	Typical storm	Reid, ⁸ Riley, ³³ Escritt, ³⁴	Tangent method
D	Hypothetical storm. . .	Munro ³⁵	
E	Critical storm	Coleman and Johnson, ³⁶	
		Jens, ³⁷ Laurenson ³⁸	
		Ross, ¹⁰ Rousculp, ³⁹	Time-area methods
		Ormsby, ¹³ Hart ⁴⁰	
		Hawken, ¹¹ Judson ¹²	

77. These methods were listed in order of increasing run-off. Method C, though not the most convenient, should give the most accurate results. If the distribution of typical storms was not known for the neighbourhood being studied, Methods B (tangent method) and D (hypothetical storm) could be used to find the limits within which the answer probably lay.

78. Methods A and B were generally used only for the prediction of peak flows, but they clearly implied that the hydrograph of run-off was given by the time-area curve, i.e. the plot of cumulative area contributing versus travel time. The instantaneous unit hydrograph corresponding to these cases was the derivative of the time-area curve (frequently referred to as the time-area concentration curve) and not a rectangle as indicated by the Author on p. 172. For uniform rainfall intensity, the discharge Q at a given time was proportional to the area contributing to the flow at outlet at that time. Hence

$$\begin{aligned}
 Q &= Cap = ia \\
 &= i \int_0^t \frac{da}{d\tau} \cdot d\tau \\
 &= \int_0^t i \frac{da}{d\tau} \cdot d\tau \quad i = \text{constant}
 \end{aligned}$$

$$\text{but } Q = \int_0^t u(0, t-\tau) \cdot i(\tau) \cdot d\tau$$

$$\therefore u(0, t-\tau) = \frac{da}{d\tau}$$

79. Thus in all variations of the rational method—cases A to E above—the corresponding instantaneous unit hydrograph was the time-area concentration curve.

80. A final comment on this section was merely a matter of nomenclature. Equation (5) on p. 171 was referred to as Duhamel's integral. Step functions and convolution

were used by Poisson in 1826 and the theorem proved by Mellin in 1896. It was also known as Borel's theorem, the principle of superposition, the convolution theorem, and the equation for complex multiplication.⁴¹ It might be less confusing not to attach a name to the theorem or integral but just to state that the run-off was given by the convolution of the effective rainfall and the instantaneous unit hydrograph, i.e.:

$$\begin{aligned} Q(t) &= u(0, t) * i(t) \\ &= \int_0^t u(0, t-\tau) \cdot i(\tau) \cdot d\tau \end{aligned}$$

and not to attach any name to the equation.

81. The Author rightly pointed out that any parameters used to characterize the response of a catchment should be stable in the sense that they should depend only on the catchment and remain unaffected by the characteristics of the rainfall producing the run-off. In this connexion—and again in his conclusions—he mentioned the peak of the instantaneous unit hydrograph and the lag between rainfall and run-off as both being suitable for use as the first parameter. Professor Dooge was of the opinion that the lag was decidedly superior to the peak of the instantaneous unit hydrograph as a choice for the first parameter. In analysis, the lag (which was a mean value), could be determined with much greater accuracy than the peak (which was an instantaneous value), and could also be derived more easily. Indeed, as the Author pointed out, the lag could be found from the rainfall and flow records without even deriving the unit hydrograph. In correlation studies unit hydrographs for a finite duration of rainfall excess could be handled just as easily as instantaneous unit hydrographs in determining the lag. In this way unit hydrographs derived by previous workers could be used readily. In contrast, the derivation of the peak of the corresponding instantaneous unit hydrograph from a finite-duration unit hydrograph was a tedious and uncertain business. A further advantage in using the lag was that the dimensionless instantaneous unit hydrograph based on the lag was a most convenient form for presentation and comparison. Professor Dooge would urge most strongly that the lag was used as the first parameter of the instantaneous unit hydrograph and that all such hydrographs were plotted dimensionlessly in terms of the lag.

82. The Author's suggestion that a single parameter be taken, both as a first step towards defining the indicial response and also towards correlating it with catchment characteristics, was an excellent one. For the reasons given earlier, the lag between rainfall and run-off was thought to be the most suitable parameter for this purpose and its use should be adopted as standard practice.

83. The choice of the second moment of the instantaneous unit hydrograph to serve as the second parameter in second order refinement might not be so advantageous. Because of the difficulty of accurately separating the ground-water, the end of the hydrographs were much more liable to error than the flow near the peak. In the second and higher moments these end portions of the hydrograph became of more and more significance. This problem required careful consideration.

84. In regard to the various methods used to correlate hydrographs with catchment characteristics, it was not clear how the methods of O'Kelly²⁰ and McCarthy²¹ were classified as one-parameter systems when they each appeared to have two degrees of freedom.

85. The complexities of the frequency aspect of the problem could not be adequately treated in a Paper covering the wider problem or in a discussion of this length. One comment, however, appeared relevant. The only comprehensive study of run-off co-efficients for urban areas was that of Horner and Flynt⁴² which was based on 20 years flow records. Horner and Flynt compared two alternative definitions of *C*: (1) The ratio of run-off to rainfall in the individual storms, and (2) the ratio of the flow of the given frequency to the independently derived rainfall intensity of the same frequency and appropriate duration. They found the latter value to be less subject to variation. This seemed to indicate that the problem of percentage run-off and the

frequency problem—parts (a) and (c) on p. 164 of the Paper—might be compensatory, at least partially, and that accuracy might be lost rather than gained by a detailed analysis of one of those factors without a similarly detailed analysis of the other. This might account for the fact that design procedures which took C as independent of the rainfall intensity and at the same time ignored the frequency problem had not led to major inconsistencies in observed sewer flows.

86. A further step forward in the direction suggested by the Author would be the derivation of the ground-water unit hydrograph, and the most probable separation between effective rain-producing surface run-off and infiltration producing ground-water run-off determined by statistical methods.

Mr E. E. Dawson stated that while reviewing the various methods their weak points should also be considered.

88. Had the Author in his researches ever found an attempt to prove that the discharge peak occurred when a particle of water from the farthest part of the catchment had reached the gauging station? Proof seemed necessary as this idea appeared to be common to the rational, the tangent, and the time-area methods. On the other hand, could the findings be accepted against these methods by the evidence of the unit-hydrograph method, § 32 and Fig. 7, when the unit-hydrograph method was itself suspect (see §§ 33 and 35)?

89. In the portion dealing with the unit hydrograph, the rainfall intensity was taken as being uniformly distributed over the catchment, thus neglecting the effect of variation in area such as occurs, for instance, in moving storms; such a variation could have a large effect on the peak discharge.

90. The Author did not specifically mention the effect of overbank spill in reducing peak discharge. In the problem of prediction of maximum discharge, if a unit hydrograph for a certain catchment had been derived from rain-storms which were not great enough to cause spill, it would seem that this unit hydrograph would not be applicable in the case of greater storms, i.e. great enough to cause spill; the method would appear to break down in such a case.

91. Regarding the controversy mentioned in § 35, would it not be possible to test the validity of this and other theories by model experiments with controlled rainfall on simplified catchments? Even qualitative results should be of value.

Mr F. V. Appleby thought that there was a great need for fundamental studies of the run-off problem in general. The Author's work in this direction was therefore welcomed and it was to be hoped that in the outcome the Author would substantiate his claim to have developed a systematic method of investigation suitable either for urban or natural catchments.

93. In his historical review the Author commented on methods of merit and of differing purpose and his critical analysis put forward some interesting points of view. Even so, much of what he had to say was tentative and inconclusive and in withholding the issue of his conclusions for a later Paper the scope for useful discussion was in this degree limited.

94. The Author evidently intended to make considerable use of the idea of indicial response (or unit hydrograph) as the recorded (or assumed known) reaction of a drainage system to a rainfall impulse of unit time, but indefinite magnitude or alternatively a rainfall impulse of unit magnitude and of indefinite time. The Author described these responses by the function $U(1, t)$, and the variable effective rainfall impulse by $i(\tau)$. He then made his principal assumption that the response of the system to a series of impulses $i(\tau)$ would be the sum of the responses to the individual unit impulses contained in $i(\tau)$ taken in order. On this assumption, and the assumption that both i and U were continuous functions, then the summation of the product of these variables by means of the Duhamel integral would produce the total hydrograph of run-off. This statement by the Author in mathematical terms of continuous functions was

identical in principle with the normal classical step-by-step technique of hydrograph determination from unit-hydrograph data.

95. The Author freely conceded that linearity was assumed without proof. Even so, it was felt that having regard to the fundamental nature of his investigation, the Author should not leave it without some fuller discussion. From his statements in § 35 the Author was obviously uneasy about the matter and it would be helpful if he could enlarge on his statement therein about the apparent conflict between unit-hydrograph theory, and the laws of hydraulics.

96. The essential part of the Author's thesis seemed to be contained in the sections covered by §§ 23–32. In § 24 there appeared to be a little confusion on the essential definition between a unit impulse and a step impulse. These were clearly shown in Figs 9 and 10 respectively. In the first case, the step was a unit of time and in the second a unit of magnitude. This was not made clear in the text. The Author then went on to say that either of these responses was known as an instantaneous unit hydrograph. Here there seemed to be a contradiction in terms. Could the Author give a physical idea of this instantaneous hydrograph concept? Considering any remote element of some small surface of ground, the run-off from the shortest pulse of rain could never have an instantaneous rate of flow in its aspect as a flood embryo. Even so, the idea that the derivative of the time-area curve (§ 31) resulting in the time area histogram as an instantaneous hydrograph was not so inaccurate as the Author supposed. A histogram of an urban area of about 100 acres with a calculated minimum time of concentration of 35 min was given in Fig. 8. A series of actual run-off hydrographs

FIG. 8

from uniform rainstorms varying in duration from $1/10t_c$ to $2t_c$ was given in Fig. 9. With the exception of record 179(b) which had 30 min. of rain of low intensity following the $1/10$ impulse, all the top line of hydrographs showed clearly the basic form of the histogram and quite clearly showed also a "band width" time, of the order of the total minimum concentration time of the network. It was seen also that the peaks of flow develop in the same sequence as the histogram.

97. The ordinates of the histogram were in effect flood-source potentials (F.S.Ps) with an implied space co-ordinate of distance through the channel network converted to time by a high-flow steady-state velocity. These flood-source "potentials" were swept by a succession of varying rainfall intensities, and the products of the F.S.Ps and the rain intensities formed a flood embryo, each being an element in the form of a hydrograph and each diffusing and merging with its neighbours by an orderly process to contribute to a growing flood "wave". Since each flood embryo was a "hydrograph"

FIG. 9

in its own right the histogram would emerge in the $1/10t_c$ impulse graph as perched on the recession tails of immediately preceding histogram ordinates.

98. As the rain impulse durations increased the form of the hydrograph changed: the form between $1t_c$ and $2t_c$ was smooth and rounded as if it had had a totally different origin from the $1/10$ graph. This was due to the change from the sequence phase when $\tau = 1/10t_c$ to the summation phase when $\tau = t_c$ or more. The commencement of the summation phase was clearly shown by the indicator spots on Fig. 9.

99. Fig. 10 showed recorded unit hydrographs from an area of 21.3 square miles, Potomac Drainage Basin, data U.S. Army Corps of Engineers,⁴³ and illustrated the same process of change in the hydrograph with impulse time as for an urban area. It would be impossible therefore to agree with the Author's inferences in relation to his Fig. 7, using the late Mr. J. J. O'Kelly's data. Superimposing the $1/10$ sequence hydrograph on the $2t_c$ graph the situation was much the same as that shown in the Author's Fig. 7. Unfortunately he did not show the relative rainfall information.

FIG. 10

100. It is hoped to persuade the Author by this demonstration that the steady-state histogram of uniform high-flood contributing area could be of considerable value in the technique of run-off analysis.

101. With these facts in mind, it was clear that the relative duration of the impulse had a very important influence on the resulting hydrograph, so that the term "unit hydrograph" needed a fuller reference which should bear some relation to the time response of the drainage basin.

102. A flood wave was a growing process in an hydraulic environment, creating for itself its own means of removal by the essential potential of its elements, a forming, organic process, not a wave, although in due course when the maximum of growth was complete further progress (movement) would come within the field of hydraulic wave calculations.

103. Approximations were invariably necessary in practice, but unless there existed a sound fundamental theory to which these might refer, there could be no satisfactory measure of validity. Flood growth, like all physical processes, involved the consideration of variation in both time and space (t, x) and eventually this must lead to fundamental statements in the form of partial differential equations.

104. Fundamental hydraulic laws followed the continuity equation $\partial q/\partial x + \partial s/\partial t = 0$, whereas practical flood-routing systems led to the fundamental diffusion equation $\partial q/\partial x - \partial s/\partial t = 0$. Herein seemed to lie the paradox⁴⁴.

Mr P. O. Wolf observed that he had unfortunately been unable to make a study of all the Author's references and could not, therefore, comment on a number of the promising suggestions made in the latter part of the Paper.

106. The Author had presented, in an admirably elegant form, a survey of the methods commonly used for determining storm run-off, against the background of the unit-hydrograph theory. He had used as his reference system a theory which, as he had pointed out so well, was based on the assumption of linearity—not once but in several respects. To that extent the unit hydrograph was an idealization or, in other words, a model which might be dissimilar to actuality to a greater or lesser extent. As Linsley² had pointed out, the unit hydrograph was mechanically an exact model only to the extent that *equal* rainfall following equal "antecedent precipitation," etc., would produce equal run-off hydrographs. The Author had, in § 56, mentioned some of the variables which would affect the relation between rainfall and run-off, such as the quantity R and the duration T of the rainfall, the variability A of the rainfall intensity, the index B of the soil-moisture deficiency, and the ground-water discharge C , and had mentioned others elsewhere, one of the most important in Mr Wolf's view being possibly the variability of the storage coefficient K (§ 47) in time and space.

107. It appeared that the Author had treated the run-off coefficient C (in equation (1)) as denoting the ratio between the volume of storm run-off and the total volume of rain R causing it, but there was another run-off coefficient C_1 (to be defined later) which had been used by experienced designers.

108. As regards the variability A of intensity during the storm, Braine had discussed⁴⁵ the importance of that factor particularly in work on storm run-off from urban areas, and Mr Wolf had proposed⁴⁶ an idealization of actual storms which tended to exhibit a distinct rise to a peak intensity, with a subsequent falling off. If the time to peak of the rainfall intensity was denoted by T_1 , the—admittedly idealized—form of the acute rain-storm could be readily altered to any value of T_1/T , and Mr Wolf hoped to be able to derive equations to show the effect of the amount of variability and of the value of T_1/T (i.e. the position of the rainfall peak) on the magnitude and position of the run-off peak.

109. The frequency $F(R, T)$ (§ 8) posed a further problem, well described by the Author, but not necessarily relevant in a discussion of the fluid mechanics of run-off. The Author was certainly correct in pointing out (in § 17 *et seq.*) that the rainfall variability (both A and the ratio T_1/T) would affect the run-off frequency F .

110. A vital problem of unit-hydrograph theory was touched on in § 35, and the

Author would greatly add to the value of the Paper if he could discuss more fully his statement that "the assumption of linearity . . . is . . . an over-simplification, . . . it is not necessarily contrary to the laws of hydraulics."

111. Turning to some points of detail, Mr Wolf thought that the time of concentration T_c (mentioned in § 8) was now recognized not to be "the time required for a particle of water to flow from the farthest point of the catchment to the gauging station." He thought that the present-day successors of Kuichling and Lloyd-Davies would call it the time required for the rainfall on the farthest point of the catchment to make itself felt at the gauging station (by wave translation before the particle had travelled the full distance), and T_c might be effectively equated with the "time to peak." That was certainly the interpretation put on it in recent years by a great authority on the subject, the late G. Bransby Williams.⁴⁷ Both the velocity of (particle) flow and the wave celerity increased with the depth in the channel, and modern use of the "rational method" allowed both for the change in the definition of T_c and for its variability with rainfall intensity R/T .⁴⁸ One of the other influences on the time of concentration, convexity of the catchment, had been discussed⁴⁶ elsewhere, and the preliminary results of further work showed that with a "hog-backed" topography the variation of T_c with R/T was especially pronounced: a result which could be incorporated in the "rational method" without undue difficulty.

112. Mr Wolf did not wish to pose as the defender of the "rational" method, which he knew had been used to provide rather irrational answers, but there were other comments on that theory in the Paper which required qualification. The Author's way of showing the family connexion (in § 27) was very attractive, but was its result, the rectangular hydrograph of Fig. 5, really so "inconceivable"? If a catchment were imagined to consist of a steep uniformly sloping paved area of uniform roughness, and if (by definition of the "instantaneous unit hydrograph") a 1-in.-thick layer of water were deposited on it instantaneously, the run-off hydrograph, except at the ends, would be represented by the rectangular pattern of Fig. 5. A uniform rain-storm lasting a time T would, by superposition of many small rectangles, lead to an isosceles trapezoid or triangle: not an unusual first approximation to a hydrograph.

113. Moreover, was the rectangular outline of Fig. 5 really the only one? It seemed that an isosceles triangle²⁰ (or a triangle on a rectangular base of any desired height so long as the total area under the "roof" represented the required unit volume) would serve as well, giving an S -curve as shown on p. 310 of Williams' chapter.⁶

114. Those speculations clearly applied to unusual catchments, and were of little value where the flow passed through channels or sewers of considerable storage capacity. Mr Wolf also recognized that the "instantaneous unit hydrograph" in the shape of an isosceles triangle was a physical impossibility in the case of the uniform slab he had just postulated; but much of the Author's elegant work seemed to be a mathematical development of the equations describing an idealized run-off model, and it seemed that it would be possible to have several variants in such a development.

115. To revert to equation (1) and Fig. 5, the run-off co-efficient could also be defined as $C_1 = Q \cdot (T_c/R)$ (where Q was the peak rate of run-off)—a definition which had been used in the determination of channel and sewer capacities, although it did not make it possible to study *volumes* of storm run-off. If C_1 (based on previous observation) were used, instead of C , to study, for example, the response of the middle catchment described on Fig. 7 (Rahan) to a 7-hour storm, the discrepancies would be much less alarming.

116. Finally, Mr Wolf wished to support the Author's conclusions, in § 59, and to express the hope that he would extend his interesting study, beyond the proposed correlation, to tests of the limits of validity of the assumptions made in the various methods.

The Author, in reply, said that most of the points made by the correspondents concerned the first part of the Paper, viz., the demonstration that the rational method, the tangent method, and the time-area methods were all particular cases of the unit-hydrograph theory and could, therefore, be compared, by comparing the instantaneous

unit hydrographs corresponding to each method. It was shown that the rational method was identical with the unit-hydrograph method provided the instantaneous unit hydrograph was taken as a rectangle of duration T_c (Fig. 5). Similarly the time-area methods corresponded to the unit-hydrograph method with the instantaneous unit hydrograph taken as the derivative of the time-area concentration curve. Professor Dooge appeared to have misunderstood this comparison since he disagreed with the demonstration that the rational method was equivalent to the assumption of linearity and a rectangular instantaneous unit hydrograph. He showed that, by making the further assumption that the effective rainfall was uniform in time, a different instantaneous unit hydrograph could be used. This was, of course, quite true but the assumption of uniform effective rainfall was not part of the unit-hydrograph theory and therefore the original statement still stood—that the rational method was identical with the unit-hydrograph method, provided a rectangular instantaneous unit hydrograph was assumed.

118. Mr Wolf's suggestion (§ 113) that a triangle or other shape would serve was unacceptable for the same reason. Mr Wolf also suggested that the rectangular instantaneous unit hydrograph was not so inconceivable except for the ends. This was possibly true, depending on how much variation one would be prepared to tolerate, but what was the reason for using a rectangle at all, when it was generally known that the shapes of all short-period unit hydrographs experienced in practice were very similar and generally of the form of Commons's basic hydrograph. There appeared to be no reason why the rational method should not be supplanted for all purposes by a simple unit-hydrograph method using a fixed basic shape and relating one parameter (the lag) to the time of concentration of the catchment as ordinarily measured. The use of a fixed shape based on experience would be better than using a rectangle which was certainly not the right shape. In the Author's opinion such a method would give better results than assuming the shape to be given by the derivative of the time-area concentration curve.

119. It was common experience that all short-period unit hydrographs resembled one another in shape much more than they resembled the derivative of the time-area concentration curve. The Author agreed that there was some resemblance between the short-period unit hydrograph shown by Mr Appleby (Fig. 9), and the corresponding derivative of the time-area curve (Fig. 8). This, however, was more likely to be because Fig. 8 was itself very similar to a typical short-period unit hydrograph. Fig. 7 appeared to have been misunderstood by Mr Appleby. The curves shown there were all instantaneous unit hydrographs, some derived empirically, and the others derived by the rational and time-area methods. It was therefore legitimate to compare them. The routing of the derivative of the time-area concentration curve to obtain an instantaneous unit hydrograph was a practice which had grown up in recent years. That this should be necessary was itself a serious reflexion of the time-area method.

120. The Author agreed with Mr Watkins that as certain factors could be more easily measured on urban than on natural catchments it might be desirable to develop different methods for the two cases. Mr Watkins had sent the Author unpublished reports on a very extensive investigation which he was making on urban areas. While it would be inappropriate to comment at length on Mr Watkins's work it was fair to say that the method was considerably more sophisticated than the simple time-area method. The application of modern methods to urban run-off calculation was long overdue and the publication of Mr Watkins's results was awaited with interest.

121. Messrs Dawson, Appleby, and Wolf had discussed the assumption of linearity. Mr Appleby suggested that the Author had confused the concepts of instantaneous unit hydrograph and S-curve. The instantaneous unit hydrograph was the response to a unit impulse (i.e. unit volume of effective rainfall of indefinitely short duration) and it was described in the Paper by $U(0, t)$. The S-curve was the response to unit step input and was (equation (4)) the integral of the instantaneous unit hydrograph. The instantaneous unit hydrograph and the S-curve were the two opposite limits of the unit hydrograph.

122. The Author agreed with Mr Appleby (§ 101) that the term "the unit hydrograph of a catchment" was incomplete without specifying the duration of the unit hydrograph. For this reason it seemed preferable to use parameters of the instantaneous unit hydrograph in a correlation rather than parameters of a unit hydrograph of finite duration. To demonstrate that the assumption of linearity was not necessarily contrary to the laws of hydraulics it was only necessary to demonstrate that for one possible catchment the assumption was exact. Considering a catchment consisting entirely of a single reservoir whose outfall was governed by the equation $Q = c_1 h^m$ (where h was the head over a weir or bed of the channel, Q was discharge, and c_1 and n were constants for the particular catchments), it could be assumed that the shape of the reservoir was such that the storage above the crest was given by $S = C_2 h^m$. Obviously if $m = n$ then $S = C_3 Q$. Such a catchment was possible, in the sense that it was not contrary to the laws of hydraulics. Because of the linear storage/discharge equation, and the equation of continuity, the operation of this catchment was described by the first-order linear differential equation $Q(t) = \frac{1}{Q_3 D + 1} i(t)$, where $i(t)$ was the input and D the differential operator d/dt . The velocity of the water issuing from this catchment varied with time, nevertheless the unit-hydrograph assumption of linearity was exactly fulfilled. It was thus seen (a) that the unit-hydrograph theory did not depend on the assumption of constant velocity at a given point, and (b) that since the particular catchment specified was hydraulically possible, the unit-hydrograph theory was also hydraulically possible. Experiments were currently being prepared at the Hydraulics Research Station, D.S.I.R., Wallingford, to test the assumption of linearity on a catchment of about $\frac{1}{2}$ acre.

123. Mr Dawson had pointed to the fact that the unit-hydrograph method made no special provision for dealing with storms non-uniformly distributed in area. There was, however, no reason, where sufficient data were available, why the catchment could not be broken up into sub-areas and the run-off from each calculated and combined. This in fact appeared to be common practice in the United States of America.

124. The Author agreed with Professor Dooge that, strictly speaking, the methods of O'Kelly and McCarthy did not imply that all instantaneous unit hydrographs having equal peaks were identical as stated in the Paper. On the other hand all model instantaneous unit hydrographs having equal peaks were identical. Therefore while the model unit hydrographs were "one parameter" a second degree of freedom was introduced by applying the Froude model law to account for variations in area.

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