Paper No. 6254

An approximate analysis for plating panels under uniformly distributed load†

by

Leslie Gordon Jaeger, M.A., Ph.D., A.M.I.C.E.

Discussion

Mr J. Clarkson had considered ⁴ the elastic behaviour of an initially dished plate panel, and calculated the pressure for formation of a plastic hinge at the edges, assuming the panel to be initially stress free. Above this pressure, yielding and further dishing would occur. Thus, by ignoring the initial stress system in a dished plate set up during the dishing by welding or lateral pressure, the elasto-plastic behaviour might be considered by purely elastic theory.

24. His analysis had been fairly complicated, and applicable to infinitely long panels with the edges clamped against rotation and held rigidly apart. The Author had now produced some very good approximate formulae for infinitely long panels and panels of finite aspect ratio. The assumptions were fairly sweeping. For example, although a parabolic shape was assumed for the elasto-plastic range, further dishing was presumed to arise from yielding at the centre of the panel (§ 8), and this would result in a deflected shape which was not parabolic. Nevertheless, in spite of such assumptions and inconsistencies, the Author had obtained excellent agreement with the existing experimental data, as in Figs 3 and 6 (it would be useful if the Author could indicate the elastic, elasto-plastic or plastic-membrane ranges for the theoretical curves in Fig. 3). Mr Clarkson considered that, before fully accepting the formulae presented, comparisons should be made with experiment for a range of panel slenderness ratios (L/h) and aspect ratios, and it was unfortunate that there was a shortage of such experimental data.

25. In practice, the sizes of the members in arrays of panels supported by a mesh of beams were such that it was unlikely that the panel edges were held rigidly apart. This observation held whether a single panel was loaded at one time as in some deck structures, or the complete array of panels was simultaneously loaded as in ship bulkheads and bottoms. The inward movement at the panel edges might significantly affect the behaviour particularly for long rectangular panels, and, so far, very little work had been carried out to investigate the restraint against inward edge movement for panels in an array. It was possible that, for many cases, approximately correct results might be obtained using the opposite boundary condition to that assumed so far by the Author and Mr Clarkson, namely that the edges of the panel were free to slide towards each other. It might be seen that much further work on flat plates, particularly experimental work, was still required.

R. M. Haythornthwaite thought that Fig. 3 offered a striking confirmation of the proposed approximate theory as applied to very thin plates. Even at high deflexion the

† Proc. Instn civ. Engrs, vol. 10, p. 137 (June 1958).

⁴ J. Clarkson, "The strength of approximately flat long rectangular plates under lateral pressure". Trans N.E. Coast Instn Engrs & Shipbuilders, vol. 74 (1958), p. 21 and D1-D8.

agreement obtained was better than with the plastic membrane theory (equation (3)). However, to ascribe this departure from the membrane theory to elastic action in the plate might be incorrect. A similar improvement could be made without introducing elastic action, by discarding the special assumption that the plate remained parabolic. An analysis by Mr Haythornthwaite (to be published) indicated the deflexion of a beam or wide plate made of rigid-plastic material to be given by

$$\frac{w}{h} = \frac{1}{2} \sqrt{\frac{(f-1)(f+1)^2}{f}}, \text{ where } f = \frac{1}{4} \cdot \frac{q}{\sigma_0'} \left(\frac{L}{h}\right)^2$$

was the ordinate in Fig. 3. In this analysis the elastic action in the beam was ignored, yet the resulting curve fitted the test data equally well for points beyond the elastic range ($f > 1$, approximately). This fact suggested that the departure from plastic membrane action observed by Young might have been due not to elastic action but to the changed shape of the plate consequent upon the presence of bending resistance.

27. A plastic membrane theory or an elastic-plastic theory built on a plastic membrane deflexion pattern was likely to give reasonable results whenever the elastic load/deflexion slope was of the same order as the plastic membrane slope, as was the case for the tests discussed in the Paper. For thicker plates, however, bending action was likely to be more important, in particular near the limit load. In the case of circular plates, E. T. Onat and the writer had developed both plastic bending and plastic membrane theories and, after a series of tests, concluded that the plastic membrane theory was quite accurate at relatively low deflexions whenever the elastic load/deflexion slope was less than about twice the plastic membrane slope⁵. At higher slopes a bending theory became essential. Had the Author tried to correlate his theory with tests on such thicker plates?

The Author, in reply, agreed with Mr Clarkson that the assumptions made were fairly sweeping and that much more experimental evidence was needed before the formulae presented could be accepted. At best, of course, the method was only approximate; there were discontinuities in behaviour at the transition from one stage to the next, and so on.

29. It was desirable to have clearly in mind just what one would expect to achieve by use of the method presented. The practical problem to be faced was that given a plating panel with a certain dish, the dish being a result of past loadings, what load would the panel stand without the dishing getting any worse? To this question it was believed that the method given in the Paper would give a fair answer, even though the assumptions made in the analysis were clearly not realized in fact.

30. Dealing with Fig. 3, it should be emphasized that the full-line curve was that of Young, the plotted circular points having been derived by use of the method presented in the Paper. It was not possible to be precise about the change points from one stage to another, because of the discontinuities already referred to. For example, on all three graphs of Fig. 3 it would be seen that, over the middle range, the plotted circular points lay on a curve which drooped slightly across Young's curve. This was the elasto-plastic portion. The effect was clearly seen in Fig. 3c where the two uppermost points were from plastic-membrane equations, and those immediately below were from elasto-plastic equations. The fact that the change-over point could not be identified precisely was not of great moment, since both sets of equations gave an acceptable approximation. Thus, for any permanent set, however large, the elasto-plastic equations could be used, the small error involved being on the safe side.

31. It would be interesting to see whether a workable approximate theory could be derived for the opposite boundary condition referred to by Mr Clarkson. Indeed,

⁵ E. T. Onat and R. M. Haythornthwaite, "The load-carrying capacity of circular plates at large deflection". *J. Appl. Mech.*, vol. 23 (1956), p. 49.

a striking feature of work on plates was the variety of simplifying assumptions which could be made without greatly altering the load/deflexion/permanent-set relation. This led to Mr Haythornthwaite's very interesting contribution, wherein a markedly different assumption of material properties made little difference to the load/deflexion relation at fairly large deflexion. However, the load/deflexion relation was not the whole story; as already stated, the main concern was with acceptable permanent set, and not only with the "straight through" load/deflexion curve (see, e.g. Fig. 6). The nature of Mr Haythornthwaite's assumption was such that elastic recovery of the plate on unloading was ruled out, so that a good estimate of permanent set could not be expected.

32. For the load/deflexion relation, Mr Haythornthwaite's future Paper would be read with much interest. The Author agreed with Mr Haythornthwaite that this method could be expected to be more accurate than that given in the Author's present Paper if the plates were very thick, and less accurate if they were very thin. No attempt had been made to correlate the Author's theory, presented in this Paper, with the behaviour of very thick plates.