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The elastic buckling of brittle columns†

by

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and

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The following contributions were received in writing.

Professor J. B. B. Owen (Professor of Civil Engineering, University of Liverpool) said that the Paper was essentially concerned with the load carrying capacity of struts which varied in section along their length. The problem was further complicated by that variation depending on the end load. Their analytical solution of the discontinuous equations of equilibrium was very neat and with their results available (as presented in Fig. 4 for example) one was tempted to embark on approximate calculations for comparison with their more exact values.

A lower limit to the strength of those struts was given by the intersection of the e_c/d curves of Fig. 4 with the vertical "cracking line" of that Figure. The intersections gave the loads at which the struts concerned just cracked at the centre.

An upper limit was obtained at the intersection of those curves, which were now shown by broken lines in Fig. 4, with the vertical line at $y_c/d=0.5$. Now the struts were hinged at the centre. For small values only of the initial bow e_c , e.g., $e_c/d=0.01$, were those limits reasonably close together?

The upper limit could, however, be replaced by an estimate of the elastic critical load of the strut, which could be obtained by energy methods. A first approximate over-estimate of that critical load was obtained by assuming a bent shape given by $y=ad \sin \pi x/l$. That gave the ratio of the critical load to the critical load of the uncracked strut as:

$$\begin{aligned} (27/4\pi) \{ & \pi \{ (1+9a^2)/2 - (19/27+9a^2)x/l \} \\ & - a(9+10a^2) \cos \pi x/l + (19/54+6a^2) \sin 2\pi x/l \\ & + (a/3)(3+5a^2) \cos 3\pi x/l - (3a^2/4) \sin 4\pi x/l \\ & - (a^3/5) \cos 5\pi x/l \} \dots \dots \dots (31) \end{aligned}$$

Here x/l defined the position of the end of the uncracked section of the strut, i.e., it was such that $ad \sin \pi x/l = d/6$. The value of the ratio of the central deflexion $y_c:d$ was a , so that (31) enabled an upper limiting curve to be drawn on Fig. 4. That placed the failing loads of the struts within much closer bounds.

A still closer upper limit could be obtained if the critical loads derived in (31) were used to construct the deflexion curves of Fig. 4 in the portions in which they were shown by broken lines.

A simpler expression for the "critical load" was obtainable by concentrating attention on the section at mid-length, and finding the end load at which the curvature and

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FIG. 13.—APPROXIMATE LOAD/DEFLEXION CURVES (EQUATION (36))

FIG. 14.—APPROXIMATE MAXIMUM-LOAD CURVE (EQUATION (38))

For an eccentrically loaded column, where the load was applied outside the middle third, the equilibrium equation (16) applied continuously over the length, and the maximum load was given without approximation by:

$$\frac{P_{\max}}{P_{\text{crit}}} = 7.65 \left(\frac{1}{2} - \frac{e_p}{d} \right)^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

This expression also gave the maximum load with fair accuracy for smaller eccentricities, although the limiting value for $P_{\max}/P_{\text{crit}}$, when $e_p=0$, was 0.95 instead of 1.0.

There was an obvious similarity between equation (39) and Professor Owen's equation (32). However, neither that equation nor his suggested empirical development gave the correct limit when $y_c = \frac{d}{6}$. It might be noted that equation (39) did not relate maximum load to initial eccentricity.