DISCUSSION

Generalised framework of limit equilibrium methods for slope stability analysis

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The authors are to be congratulated on collecting several different methods that have been used for slope stability analysis.

The writer finds it interesting that the authors multiply the value of the total normal stress across the slip surface, $\sigma_0(x)$, by a modifying function as in equations (6) and (7) in order to satisfy the equations of equilibrium. It seems that equation (7) might be more useful than equation (6). The real test of whether the total normal stress across an irregular slip surface has the correct distribution will be when the same factor of safety is obtained if one changes the position of the point about which moments are taken. Perhaps the authors have achieved this requirement.

However, the whole method proposed by the authors hinges on their statement ‘[W]e can initially assume a normal stress distribution $\sigma_0(x)$...’ (see p. 379). A great deal of effort and understanding is required to use equation (31) to estimate the initial distribution of $\sigma_0(x)$.

Certain methods have not been covered by the authors. The general method presented by the writer (Sparks, 2000) yields results similar to the best traditional method, namely the complicated rigorous method by Janbu. Accurate results can be obtained by using very few slices.

A self-converging algorithm hastens the iterative process. Usually three cycles of calculations are sufficient to obtain a good answer.

Consider the non-circular slope shown in Fig. 10, which is the same as that shown by the original authors in their Fig. 8. The slices are numbered from the upper left region down to the toe of the slope at the bottom right. The forces on a typical slice are shown in Fig. 11. The total weight, W , of each slice includes any surface loading, or the weight of any free water on the surface of the soil.

A trial factor of safety is assumed for the first cycle of calculations, and in this cycle the stability of each slice is considered in turn, starting with the upper left-hand slice 1. The following equations (75)–(80) are used in this order for each slice, and this yields the force P_R as the output from these equations for this particular slice. This force, P_R , becomes the force P_L in the next slice.

Fig. 10. Non-circular slip surface, using five slices

Fig. 11. Typical forces on a slice

$$\tan \phi'_d = \frac{\tan \phi'}{F_s} \quad (75)$$

$$X = W_{\text{horiz}} + U \sin \alpha + U_L + P_L (\cos \delta_L) - U_R - \left(\frac{c \cdot \Delta L}{F_s} \right) \cos \alpha \quad (76)$$

$$Y = W + P_L (\sin \delta_L) - U \cos \alpha - \left(\frac{c \cdot \Delta L}{F_s} \right) \sin \alpha \quad (77)$$

$$D_1 = \cos(\alpha - \phi'_d) \cdot \cos \delta_R + \sin(\alpha - \phi'_d) \cdot \sin \delta_R \quad (78)$$

$$P_N = \frac{(Y \cos \delta_R - X \sin \delta_R) \cdot \cos \phi'_d}{D_1} \quad (79)$$

$$P_R = \frac{Y \sin(\alpha - \phi'_d) + X \cos(\alpha - \phi'_d)}{D_1} \quad (80)$$

The last slice (bottom right) abuts the atmosphere, and the fictitious force P_R between the last slice and the atmosphere should be zero if the correct value of the factor of safety has been assumed. A compressive value of P_R for the last (lower) slice indicates that F_s must be reduced for the next iteration. This method satisfies two equilibrium equations: the horizontal equation of force equilibrium, and the vertical equation of force equilibrium for each slice.

The equation for the new factor of safety, F_s , for the next cycle is

$$F_s = F_{s0} \left(1 - \frac{P_{Rn}}{\Sigma W \sin \alpha} \right) \quad (81)$$

where F_{s0} is the previous (i.e. current) factor of safety, and P_{Rn} is the compressive value of horizontal P_R for the last (n th) slice: that is, P_{Rn} will have a zero value when the correct F_s is used.

Table 5 shows the input values (SI units) for the non-circular slip surface shown in Fig. 10. Note that only five slices have been considered in the Sparks method, whereas the original authors have apparently used 100 slices for some of their calculations. The values of the angles δ_L and δ_R can be estimated from a trial line of thrust, but the writer prefers to use the average value of the slip angles, α , for the two adjacent slices.

For the non-circular slip in Fig. 10, if we start with an initial trial value of unity for the factor of safety, then after one cycle the next trial value becomes 1.314; after two cycles the next value becomes 1.300; and the final solution value is 1.301. We can compare this with the value of 1.299 quoted by the authors who used the rigorous Janbu method, and many slices.

In the case of the circular slip circle shown in Fig. 12, we have used only six slices in the Sparks method. In this example, the given values are $c' = 600 \text{ lb/ft}^2$ and $\phi' = 20^\circ$. Starting with an initial trial value of unity for the factor of safety for this circular slip surface, after the first cycle the next trial value becomes 1.817; after two cycles the next value becomes 1.787; and the final solution value is 1.789. We can compare this with the values of 1.776 or 1.836 quoted by the authors who used the rigorous Janbu method and many slices for the circular slip surface.

It seems that the Sparks method can compete with the best method (namely the Janbu method) and will yield similar values for the factor of safety.

Authors' reply

The authors thank Dr Sparks for his interest in the paper. The purpose of the paper was to present a generalised framework incorporating the currently used methods of slices. The proposed framework features a new algorithm for calculating the factor of safety of slope based on assumptions regarding normal stress distribution over the failure surface. If an initial normal stress $\sigma_0(x)$ is specified, then we can readily obtain solutions for factor of safety that satisfy one of four combinations of equilibrium conditions, as given by equations (20), (23), (26) and (27) in our paper, respectively. However, the conventional methods of slices feature

Fig. 12. Circular slip surface, using six slices

assumptions about interslice forces rather than about normal stresses over the slip surface. The general form of assumptions about interslice forces is given by equation (29), which is then transformed into a general form of expression for initial normal stress $\sigma_0(x)$ as given by equation (31), with an initially assumed, or subsequently modified, value of factor of safety. Each method of slices has a specific assumption about interslice forces and specifically satisfies one of the four combinations of equilibrium conditions. To evaluate the factor of safety of slopes by any individual conventional method of slices, with an initially assumed value of factor of safety F_{s0} , and, if any, a value of the scaling factor λ , the initial normal stress $\sigma_0(x)$ is then computed according to equation (31). This results in a modified value of factor of safety. This process must be repeated until a converged value of factor of safety is obtained. This is essentially identical to the factor of safety associated with the specific conventional method of slices being used, as the value of factor of safety is dependent only upon the assumption made about interslice forces and equilibrium conditions satisfied, irrespective of the algorithms used for computation. As a matter of fact, any normal stress $\sigma_0(x)$ initially assumed, or directly computed by using the finite element method, can readily yield a value of factor of safety that satisfies the required equilibrium conditions.

The method proposed by Dr Sparks (Sparks, 2000) belongs to the HV group of methods of slices, which satisfy the horizontal and vertical force equilibrium conditions. The basic assumption underlying this method is that the inclination of the interslice force is equal to the mean slope of the bases of two adjacent slices. When an infinitesimal width of slice is chosen, this assumption is identical to the following equation:

$$T(x) = s' \cdot E(x) \quad (82)$$

This assumption is not regarded as reasonable if the inclination of segments of the slip surface is too steep or of negative magnitude. It has therefore not been widely em-

Table 5. Input values used for the five slices of non-circular slip in Fig. 11

	Slice number				
	1	2	3	4	5
Angle α under centroid	50	31	0	0	-26
Total weight, W : kN/m	564.0	1136.5	1441.0	65.7	26.3
Length, ΔL , for cohesion: m	10.82	8.23	15.32	3.05	2.74
Force, U , under slice: kN/m	0	163.3	327.2	34.9	15.7
Force, U_L , on left: kN/m	0	0	54.7	6.5	6.5
Force, U_R , on right: kN/m	0	54.7	6.5	6.5	0
Angle, δ_L , on left of slice	0	40.5	15.5	0	-13
Angle, δ_R , on right of slice	40.5	15.5	0	-13	0*
Friction angle, ϕ' , under slice	20	20	10	10	20
Cohesion, c' , under slice: kN/m ²	28.279	28.279	0	0	28.279

*Suggest use zero angle for angle δ_R for last slice (i.e. horizontal P_{Rn}).

ployed in practice. However, it is very interesting to note that this assumption automatically satisfies the moment equilibrium condition in the cases of no seismic forces and no surcharge. Neglecting the seismic force and surcharge terms, the moment equilibrium equation (52) for an infinitesimal slice reduces to

$$T(x) = E(x)y'_t + h_t E'(x) \quad (83)$$

As $y_t(x) = s(x) + h_t(x)$, the above equation becomes

$$T(x) = s' \cdot E(x) + (h_t E)' \quad (84)$$

By comparing equation (84) with equation (82), it follows that the above assumption is identical to assuming $h_t(x) = 0$: that is, the line of thrust coincides with the slip surface, and

the moment equilibrium condition is automatically satisfied. Thus it is understandable that the corresponding value of factor of safety is frequently in fair agreement with those obtained by other rigorous methods of slices. Obviously, the assumption $h_t(x) = 0$ is not acceptable from the point of view of slope failure mechanism. Therefore the more rigorous methods of slices, such as the Morgenstern–Price method (Morgenstern and Price, 1965), are still recommended in practical use.

REFERENCE

- Sparks, A. D. W. (2000). Methods for general stability analysis. *Proc. 8th Int. Symp. on Landslides, Cardiff*, 1373–1378.