DISCUSSION

An elasto-plastic model for unsaturated soil incorporating the effects of suction and degree of saturation on mechanical behaviour

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The authors should be commended for presenting such an innovative, elaborate elasto-plastic framework for predicting constitutive unsaturated soil response under isotropic stress states. One of the key aspects of the work is the authors' deep knowledge of the physical interactions that lie behind the mechanical behaviour of unsaturated soils. The proposed constitutive model and their associated conceptual framework provide an important improved understanding of soil behaviour. The present review is intended to offer some comments that further highlight the relevance of the work and its potential for further refinement and tuning.

The objective of this review is twofold:

- (a) to propose an alternative numerical procedure for obtaining the analytical expression for the assumed fundamental assumption
- (b) to discuss the effectiveness of the proposed model with regard to the 'direct' relationship between the mechanical and hydraulic responses of unsaturated soils for irreversible stress paths.

Inspection of Fig. 6 suggests that, for a suction state of $s_2 = 200$ kPa, the experimental values of the e/e_s ratio are larger than those predicted by the model equation (4), for any given value of the bonding factor ξ : that is, the experimental values fall above the curve predicted by the model equation, whereas for other suction levels ($s_1 = 100$ kPa and $s_3 = 300$ kPa), the opposite is observed. A similar observation can be made from the data shown in Fig. 8: for a suction state of $s_2 = 200$ kPa (Series I), the experimental data are larger than those predicted by the model equation.

In light of the above observations, and given the difficulties involved in measuring the interparticle forces induced by a water meniscus, as well as in determining the analytical expression for the function f , the following alternative procedure is proposed. The main goal, based on reported experimental data of normal compression lines in Fig. 4, is to determine simultaneously the analytical expression of the model equation (4) and the values of the function f for suction levels s_1 , s_2 and s_3 . To accomplish this, three parameters are considered for the values of the function f :

$$c_1 = f(s_1), c_2 = f(s_2) \text{ and } c_3 = f(s_3)$$

and another parameter a for the model equation, which is now considered in the following expression:

$$1 - a\{1 - \exp[f(s)(1 - S_r)]\}$$

With the same set of experimental data and the same procedure utilised in the paper, the best-fit values of the proposed parameters a , c_1 , c_2 and c_3 can be obtained. This procedure yields a model equation (containing the above expression) and a function f (by extrapolating for the rest of suction values) with a slightly larger computational cost, but one that fits slightly better the reported experimental data.

Based on equations (2), (3), (4) and (6), equation (5) can be rewritten as:

$$e(p, s) = 1 - a[1 - \exp\{bf(s)[1 - S_r(p, s)]\}] \times \{N - \lambda \ln[p + S_r(p, s)s]\} \quad (13)$$

where $e(p, s)$ is the void ratio corresponding to the isotropic stress state (p, s) , p is the isotropic net stress, s is the matric suction, $S_r(p, s)$ is the degree of saturation for the stress state (p, s) , a and b are the model parameters, and N and λ are the parameters of the normal compression line for $s = 0$.

Inspection of equation (13) suggests that a 'direct' relationship between the mechanical behaviour (measured by the void ratio $e(p, s)$) and hydraulic behaviour (measured by the degree of saturation, $S_r(p, s)$) can be established for irreversible unsaturated soil stress paths. So, apart from being crucial for establishing the state variables of the constitutive model (p'' , ξ) and the yield locus expression (equation (10)), the degree of saturation also determines the value of the void ratio.

Specifically, equation (13) is a two-way relationship between the values of the void ratio and the degree of saturation (contrary to common practice in other constitutive models that use differential expressions of the void ratio) and both functions being evaluated at the same isotropic stress state (p, s) . Taking into account that the most important phenomena and characteristics of unsaturated soil usually correspond to irreversible behaviours, equation (13) can be considered as a model of the degree of saturation, on the stress plane (p, s) , for irreversible soil behaviour. That is, taking the values of the parameters a , b , N and λ from Table 1, and the values of the function f from Fig. 3, equation (13) predicts the behaviour of the degree of saturation on the plane (p, s) , from the results of the experiments performed by Sharma (1998) (Figs 12–17), for the paths corresponding to irreversible soil behaviour.

The above leads us, on the one hand, to look for 'equivalences' between the irreversible phenomena of the mechanical and hydraulic behaviours. In this respect, Wheeler *et al.* (2003) arrive at important results, because, among many other things, they include hydraulic hysteresis characteristics in the yield locus. On the other hand, it leads us to look for models of the degree of saturation in the whole isotropic stress plane (p, s) , including, for example, the experimental fact according to which, at constant non-zero suction, isotropic compression saturates the soil, thus being able to determine the shape of the stress positions on the plane (p, s) corresponding to the saturated condition.

Authors' reply

The authors thank the discussers for their comments on the paper. They refer to two different issues. First, a refinement of the model is proposed that aims at better reproducing the experimental data presented in Fig. 6. The discussers rightly identify the importance of this relationship

for the model, and, in that respect, their effort to provide an improved fit between model and experimental data is commendable. However, the added complexity (three extra parameters are introduced in order to fit exactly three experimental curves, which in fact transforms the model into a fitting procedure) is questionable, given the accuracy already exhibited by the original model (see Figs 6 and 7). Moreover, considering the existence of experimental uncertainties, the gain from such added complexity is, in the authors' opinion, negligible in comparison with the drawback of precluding an economical assessment of parameters.

In the second part, the discussers draw attention to the fact that equation (5) is a state equation linking the degree of saturation S_r , the void ratio e , the mean net stress $\bar{p}$ and the suction s for an unsaturated soil element undergoing irreversible mechanical straining. They indicate that equation (5) allows the expression of either e as a function of S_r , $\bar{p}$ and s , or S_r as a function of e , $\bar{p}$ and s . The latter form of equation (5) is referred to by the discussers as a 'model for the degree of saturation', and it is regarded as a water retention model for the unsaturated soil.

The authors disagree with this statement, because the variable s cannot be considered as an independent variable; it is linked to S_r by an additional independent (though non-unique, if hysteretic) relationship: the water retention curve. The response of unsaturated soil is therefore clearly determined by two 'constitutive' equations: the mechanical model and the water retention curve. This duality expresses the fact that variation of degree of saturation due to mechanical straining and to changes in water content are two different physical phenomena that should be modelled independently and further coupled in a full hydro-mechanical model. The model described in the paper focuses on the mechanical

response of unsaturated soils, and does not include any hydraulic aspect.

However, the authors would like to add some additional comments on the prospect of coupling the proposed mechanical model with the water retention curve, which constitutes one of the final objectives of the present work. Edgar (1993), Wheeler (1996), Dangla *et al.* (1997), Houlsby (1997), Vaunat *et al.* (2000) and Wheeler *et al.* (2003) have provided contributions to such a goal, rooted in the concept of work input due to mechanical and hydraulic actions. Houlsby (1997), in particular, demonstrated that the following alternative sets of work-conjugate variables can be used as controlling hydro-mechanical variables:

- net stresses and strains plus suction and change in volumetric water content.
- average skeleton stresses and strains plus product of suction by porosity and change in degree of saturation.

On the basis of the first set of variables, Vaunat *et al.* (2000) proposed describing the volumetric response of an unsaturated soil by means of four planes: net mean stress, p , against suction, s ; the compression plane $p-e$; the retention curve plane $s-e_w$ (e_w is defined by the volume of water in a volume of soil containing a unit volume of solid); and the $e-e_w$ plane. An example of such a representation for the case of a suction-controlled isotropic loading test performed on a plastic clay is presented in Fig. 18. It is possible to observe in this figure the path followed by the degree of saturation $S_r = e_w/e$ during the applied mean stress–suction path. A zone of small and largely reversible changes in degree of saturation can be observed during the early stage of first wetting (AB), the early stage of the first drying path (BC), and during the following suction cycles. Large and irreversible changes in degree of saturation are observed in

Fig. 18. Four-planes representation of experimental results obtained during an isotropic loading suction-controlled test on Boom clay ($w_L = 56\%$, $w_p = 29\%$). From Vaunat *et al.* (2000). Data are from Romero (1999)

the later stages of paths AB and BC, when the soil reaches the main wetting curve and the main drying curve respectively. During the following cycle (CDE), a soil element moves along scanning curves that span the space between the two main retention curves. Along these paths, changes in water content are small and can be considered, in a first approximation, as reversible.

The main wetting and drying curves are not unique but depend on the accumulated plastic volumetric strain. For example, the occurrence of plastic volumetric compression during path BC shifts the main wetting curve towards lower values of e_w , and this is confirmed by the fact that the scanning curve CD crosses the initial main wetting curve AB.

In that representation, the retention curve can be considered as the hydraulic counterpart of the compression curve, where the main curves play the role of the virgin compression line and the scanning curves that of the unloading/reloading lines under hydraulic actions. Hydraulic hysteresis can thus be seen as an additional elasto-plastic mechanism in the soil response. Application of this idea to the alternative set of variables (p'' , $s \times n$) (n being the porosity), which has been used in the paper to provide a physical explanation of the effect of degree of saturation on

the mechanical response of the material, constitutes the basis of the coupled hydromechanical constitutive law for unsaturated soils currently being developed by the authors.

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