DISCUSSION

Relationship between K_0 and overconsolidation ratio: a theoretical approach

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The authors have proposed a new theoretically based expression for determining K_0 in overconsolidated clays, claiming that ‘current approaches for evaluating K_0 are all empirical’. A single test value (Fig. 7) is used to validate the new model in comparison with other approaches. In truth, several prior theoretical approaches have addressed the K_0 –OCR relationship from both analytical and numerical perspectives. The proposed approach does not provide a direct assessment of K_0 , and seems rather complex with the different types of laboratory test required and a particularly high sensitivity to the input parameters of one-dimensional and isotropic swelling indices. The direct laboratory approaches of Garga & Khan (1991) and Abdi & Garga (1996) appear better suited for implementation, and have been more extensively calibrated than this new proposed method.

For the simple case of load–unload causing mechanical overconsolidation in soils, prior numerical solutions using constitutive models have shown that K_0 increases with OCR and is influenced by other parameters (e.g. Pender, 1978; Mróz *et al.*, 1979; Whittle, 1993). If the authors are apparently referring to closed-form expressions for representing K_0 during unloading, then two analytical solutions will be reviewed here (Mitachi & Kitago, 1976; Průška 1973).

The parameter representing the (negative) slope of the undrained stress path in $\sigma'_v - \sigma'_h$ stress space can be alternatively defined in the authors’ equation (14) as a positive number (γ_k). This is a common convention in geotechnics

(e.g. the virgin compression index, C_c). Then the value of lateral stress coefficient can be written directly as

$$K_{0oc} = \frac{1}{\gamma_k} [(1 + \gamma_k \cdot K_{0nc}) \cdot OCR^{(1-\chi)} - 1] \quad (15)$$

where $\chi = \kappa_{1D}/\kappa_{1S}$ = ratio of one-dimensional swelling index to the isotropic swelling index (κ defined in a natural logarithm scale). This relationship bears a remarkable resemblance to the form proposed by Mitachi & Kitago (1976):

$$K_{0oc} = \frac{1}{2} [(1 + 2K_{0nc}) \cdot ONC^{(1-\chi)} - 1] \quad (16)$$

The apparent value of $\gamma_k = 2$ in this earlier formulation does not fall within the expected values cited by the authors, $0.7 \leq \gamma_k \leq 1.0$, yet both methods can be applied to data on overconsolidated Haney clay that was subjected to a fairly extensive laboratory testing program (Campanella & Vaid, 1972). For Haney clay, one-dimensional and isotropic consolidation/swelling curves give the secant values of $0.041 < C_{s1} < 0.066$ and $0.055 < C_{sISO} < 0.085$ from the point of unloading (defined in log base 10), with corresponding $0.7 < \chi < 0.8$. Fig. 9 shows that equation (15) gives a reasonable fit for $\phi' = 30^\circ$ (defined at maximum obliquity) using the same parameters as used by the authors for Belfast Upper Boulder Clay ($\gamma_k = 0.7$; $\chi = 0.8$). The Mitachi & Kitago expression gives a similarly good fit for $\chi = 0.7$. Unfortunately, both methods suffer from the fact that they are very sensitive to the input values of χ . In fact, Mesri *et al.* (1979) showed that the determination of swelling indices

Fig. 9. Theoretical K_0 –OCR relationships showing high sensitivity of the parameter $\chi = \kappa_{1D}/\kappa_{1S}$, and comparison with data from Haney clay reported by Campanella & Vaid (1972)

in both one-dimensional and isotropic conditions is not particularly straightforward, made difficult by the non-linearity of these indices and their dependence upon OCR. Nevertheless, these methods show the importance of mechanical unloading and increase of K_0 with overconsolidation ratio for un-aged, uncemented, and unstructured soils.

Additional insight into the factors affecting the lateral stress state in soils is afforded through an independent theoretical expression proposed by Průška (1973):

$$K_{0oc} = \frac{\sqrt{K_A} \cdot OCR}{1 - K_A \cdot (1 - OCR)} \quad (17)$$

where $K_A = (1 - \sin \phi') / (1 + \sin \phi') =$ Rankine active coefficient. Fig. 10 shows that the combined effects of OCR and effective stress friction angle (ϕ') on the magnitude of K_0 during unloading are evident, although the trending match with data from Haney clay is only fair at higher OCRs. The dependence of K_0 on ϕ' was also noted by Schmidt (1966) based on the experimental data of Brooker & Ireland (1965) and now used in the authors' Fig. 8. From a larger compilation of laboratory data, these facets can be represented by the empirical expression

$$K_{0oc} = (1 - \sin \phi') OCR^{\sin \phi'} \quad (18)$$

The agreement with equation (18) and the Haney clay data are evident in Fig. 10. In review of both Figs 9 and 10, it is noted that the authors' expression is so sensitive to the input ratio of $\chi = \kappa_{1D} / \kappa_{1S}$ that a variation of only 0.05 provides more change in the evaluated K_0 than does a 10° change in ϕ' using equation (18).

In more recent studies, equation (18) gave good estimates when compared with in-situ K_0 measurements by self-boring pressuremeter, spade cells, and hydraulic fracturing in northern Europe (Mayne & Martin, 1998) and well-instrumented stress path triaxial tests on worldwide clays (Watabe, 1995), yet underestimated field K_0 values in the structured marine clays of eastern Canada (Hamouche *et al.*, 1995). For the latter, an average exponent $\alpha_k = 0.98$ was found appropriate. The effects of cementation, ageing, cyclic loading, desiccation and fabric have been researched in recent times, yet the mathematical quantification and representations in simple

closed-form analytical and numerical simulations are to be appreciated in future works.

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The horizontal geostatic stress ratio (K_0) has long been recognised as an important starting point for realistic soil–structure interaction analyses, and the authors have suggested a theoretical approach to estimating its value based on parameters from standard laboratory tests. This discussion suggests that two important aspects of soil behaviour are missing in the model used. Further, the data used for verification come from laboratory studies in which the missing physics will largely not show up. The geostatic stress ratios predicted by the method proposed in the Note, while preferable to the Mayne & Kulhawy (1982) equation, will nevertheless tend to be lower bounds to actual in-situ values.

The missing aspects of the theoretical model used by the authors are yield in unloading and cyclic effects. Referring to Fig. 3 of the Note, the authors suggest that perfect sampling moves a soil to point C on their figure with the soil remaining elastic (on an 'elastic wall' in the Cambridge School jargon). Purely elastic behaviour in unloading is certainly a usual assumption, but unloading need not imply elasticity. Isotropic softening by yielding during unloading was first described by Drucker & Seeger (1987). Moreover, generalising critical-state soil mechanics using the state parameter necessarily requires the introduction of limited hardening, and this in turn gives an internal yield cap for self-consistency. Such an internal cap predicts that yielding can arise in unloading. Although this may seem surprising, these theoretical predictions of yield in unloading are found in experiments on sands with good predictions from the simplest form of NorSand (Jefferies, 1997). Similar conclusions could be expected from 'bubble' type nested yield surface models under some conditions.

Of course one might argue that the Note was about clays, not sand, but this misses the point that predictions of yield in unloading are from theoretical plasticity and are not restricted to sands. Elastic unloading is not a necessary result of idealising soil behaviour in a plastic framework.

The second missing aspect of the theoretical treatment offered is cyclic loading, and cyclic principal stress rotation

Fig. 10. Theoretical K_0 –OCR relationships of Průška (1973) and semi-empirical power function compared with Haney clay data

in the most general sense. The model used to predict K_0 is for a simple plastic monotonic loading to a maximum stress condition followed by a monotonic unloading. Although this well matches the laboratory procedures in instrumented oedometer tests, it is a substantial simplification of many depositional environments. Cyclic loading from wave action in shallow water, cyclic strains from earthquakes, and seasonal changes in mean effective stress will all affect the soil, even under conditions of no horizontal strain. In addition, there is creep or secondary consolidation.

These objections might seem to be minor theoretical points, given the impressive match of theory to test data presented in Fig. 8. But the test data shown are for the particular case of laboratory incremental measurements under conditions of no lateral strain with fixed principal stress directions—which is exactly what the theory covers. Neither these data nor the proposed theory includes the important geologic factors affecting in-situ geostatic stress. As an example, Fig. 11 shows data from Youd & Craven (1975) on how 100 cycles of low-level cyclic shear strain nearly double K_0 . These data are for sands, and clays would usually be expected to have an attenuated response to cyclic strain. However, over geologic time, thousands or even millions of cycles might occur (depending on the depositional environment). Similar trends to those shown in Fig. 11 should be anticipated for all soils.

There are now several general constitutive models for soil, including cyclic loading, that can be directly integrated on a no-radial-strain path, with cycles of simple shear imposed, to give improved theoretical predictions of K_0 , and which can replicate Youd & Craven's data. But any of these models begs the question: just what did happen over geologic time?

The alternative to trying to guess geologic history is simply to determine its effect by measuring K_0 in situ. This is readily done for clays and silts with a self-bored pressuremeter (SBP), and can be done in sands with the same instrument, although rather more effort and diligence is required. Of course, the objection that is always raised with the SBP is the effect of boring disturbance on the inferred K_0 value. These objections have some validity for many 'methods' of K_0 determination from SBP data in the literature, but miss the point that the SBP is amenable to theoretical analysis. The effects of finite pressuremeter geometry and disturbance do not produce much uncertainty in the estimated K_0 , provided that the SBP data are analysed by

Fig. 11. Effect of cyclic straining on geostatic stress ratio (after Youd & Craven, 1975)

proper iterative forward modelling of the complete expansion and contraction data (Jefferies, 1988; Ferreira & Robertson, 1992; Pye, 1995; Jefferies & Shuttle, 1995; Shuttle & Jefferies, 1995, 1996). The key is the contraction curve of the SBP, as these additional data substantially constrain the parameter estimates, almost regardless of disturbance.

Figure 12, from Jefferies *et al.* (1987), shows some examples of the relationship between K_0 and OCR based on SBP data for K_0 and oedometer tests on adjacent undisturbed samples for OCR. The upper-bound line to equation (14), taken from Fig. 8, is shown in comparison to these data. The line shown after Mayne & Kulhawy (1982) is the same relationship as discussed in the Note, and is very similar to the lower bound to equation (14) given in Fig. 8. Although the authors' proposed theoretical relationship is a clear improvement of the empirical one suggested by Mayne & Kulhawy, nevertheless more than half the measured in-situ data lie above the upper bound of the proposed model.

To some extent the data shown in Fig. 12 are a little unusual in that they are from predominantly rock-flour-type silty clays deposited in a shallow water transgressive marine environment—both factors that will accentuate the cyclic loading aspects of soil response. But this does not affect the basic conclusion that it is not sufficient to model K_0 by monotonic confined compression of an elastic-plastic model. Theoretical models including cyclic loading and creep are interesting in themselves for the insight they offer, but it is unclear to what extent the geologic history will ever be captured in simple and easily measured parameters that can be used as model input for realistic predictions—and whether such an approach will be easier or cheaper than in-situ testing. Looking at the in-situ data in Fig. 12 it is clear that, for all practical purposes, K_0 and OCR are independent variables. As SBP testing is not overly difficult or expensive, it is suggested that the best approach is to use the SBP diligently, measure the contraction stage of the test, and properly evaluate the data obtained using formal modelling.

Authors' reply to P. W. Mayne and F. H. Kulhawy

The authors appreciate very much the interest shown by Mayne and Kulhawy in the proposed new relationship for the determination of K_0 for overconsolidated clays. The interest of these, and other, discussers reflects the fundamental importance of the topic. It will be remembered that the new method has been developed for samples of natural clay after 'perfect' sampling. This restriction was specifically introduced to reflect the linearity, reversibility, anisotropy, structure and ageing that are common in natural clays, but which are absent from disturbed or reconstituted samples.

The discussers suggest that the proposed approach is complex, and comment that several previous theoretical approaches have addressed the problem from both analytical and numerical perspectives. In view of the importance and difficulty of the problem, we do not agree that our approach is particularly complex. The parameters involved can all be obtained from standard tests routinely carried out in many commercial laboratories, although it is acknowledged that difficulties arise in the assessment and use of the parameter χ . We were unaware of the two papers (Průška, 1973; Mitachi & Kitago, 1976) cited in the discussion. The significance of these approaches in relation to the present method is discussed here.

Equation (14) in the paper can be rewritten in the form

$$K_{0oc} = \frac{1}{\gamma} [1 - (1 - \gamma K_{0nc}) OCR^{(1-\chi)}] \quad (19)$$

Since the parameter γ was naturally derived from widely accepted cross-anisotropic theory (Graham & Houlsby, 1983), we see no particular merit in changing its sign, as

Case	Site	Soil unit	Case	Site	Soil unit	Case	Site	Soil unit
1	Amauligak A-23	A2	7	Amauligak A-23	A2	13	Sauvrak G-55	B3
2	Amauligak F-65	A2	8	Sauvrak F-45	B1	14	Tarsiut P-45	A
3	Amauligak F-24	A1	9	Sauvrak F-45	B2	15	Tarsiut P-45	B1
4	Amauligak F-24	A2	10	Sauvrak F-45	B3	16	Tarsiut P-45	B2
5	Amauligak F-24	D	11	Sauvrak G-55	B1	17	Tarsiut P-45	B3
6	Amauligak I-65	A1	12	Sauvrak G-55	B2			

Fig. 12. Independence of K_0 from OCR (after Jefferies *et al.*, 1997)

suggested by the discussers in their equation (15). Mitachi & Kitago (1976) derived an expression (their equation (16)) that, using our nomenclature, can be written in the form

$$K_{0oc} = \frac{1}{2}[(1 + 2K_{0nc})OCR^{(1-\gamma)} - 1] \quad (20)$$

The basis of their approach is not clear to us. The basic assumption used in their derivation is not explicitly stated, but appears to be that if the results of a one-dimensional consolidation test are plotted on the $p'-v$ plane (that is, if the values of K_{0oc} are known for the overconsolidated clay) then the resulting graph will coincide with the results of an isotropic consolidation test. There is considerable evidence that this assumption is not correct.

The authors' approach examines the problem of K_0 from basic principles that we feel are relevant to the behaviour of natural clays. The similarity between equations (15) and (16) is possibly caused by the various unloading lines in the derivation having similar forms. It is interesting that equation (20) can be obtained from equation (19) by the substitution of $\gamma = -2$, implying that the material is isotropic. This of course has no effect on the basic but questionable assumption made by Mitachi & Kitago (1976) in deriving equation (20). We are unable to comment on the relationship suggested by Průška (1973) since no derivation is given.

The new approach has been assessed using existing data

of which the authors were aware, obtained on clay for which the values of χ and γ are known. These data have been previously compared with many other approaches, and significant agreement was found (Doran *et al.*, 2000). The results of Campanella and Vaid (1972), obtained from a careful series of tests on Haney clay, are extremely interesting. They lie within the zone defined by equation (14) and shown in Fig. 8. They are consistent with all other data available to us, and provide additional confirmation of the merit of the proposed method.

Authors' reply to M. G. Jefferies

The authors also thank Dr Jefferies for his valuable comments. As he remarks, the assessment of in-situ stresses is a complex problem influenced by several factors. The present theoretical approach based on anisotropic elasticity is certainly a considerable simplification of the prolonged history of many natural clays, and in particular it ignores the likely effects of cyclic loading. The authors consider that these and other effects will probably be included in the value of the parameter γ , which is closely associated with soil stiffnesses. This supposes, again, that sampling and trimming are indeed 'perfect' and really do represent the behaviour of the natural clay.

We are interested in the discussers' comments on yielding during unloading, which he deals with in more detail in

Jefferies (1997) in relation to sands. We agree that the problem of measuring K_0 *in situ* at great depth is now receiving more attention (e.g. Doran *et al.*, 2000). Indeed, before receiving this discussion, we have recently introduced the idea of yielding on unloading as a reason why it may be impossible to evaluate reliable preconsolidation pressures and undrained shear strengths in soft, deep, normally consolidated deposits. Dr Jefferies' comments on the use of the SBP in this context are of considerable interest.

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