DISCUSSION

Deformation localization in cavity inflation experiments on dry sand

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It seems that the softening branch of the stress–strain relationship for Ottawa sand ($D_{50} = 0.72$ mm) shown in Fig. 10(a) represents the local relation within a shear band. Using equation (5) this relationship is shown in Fig. 16 as curve 1. Judging from the large plastic shear strain γ^p needed to reach the residual state (i.e. larger than 0.5 or 50%), this relation is not likely to be the average stress versus average strain relation for a mass with some dimensions which are much larger than the shear band width (e.g. for a plane strain compression specimen). We find this curve difficult to accept, because the residual stress ratio $\mu_{res} = \sin \phi_{res}$ is extremely low—less than 0.1, which means that the residual angle of friction ϕ_{res} is less than 6° (or even zero according to equation (5) and the values of the parameters given in the Paper). Why was a relation with such low residual strength chosen to match the cavity inflation test result (Fig. 12)?

Drescher, Vardoulakis & Han (1990) and Han & Vardoulakis (1991) showed several relationships between the stress ratio μ averaged for a shear band and the accumulated shear displacement across the shear band u_s . These relations were determined experimentally from their drained biaxial tests (i.e. plane strain compression tests) and the residual angles $\phi_{res} = \arcsin [(\sigma_1 - \sigma_3)/(\sigma_1 + \sigma_3)]_{res}$ were never less than 6° . For St Peter sandstone sand with $D_{50} = 0.165$ mm, ϕ_{res} is 33° (from fig. 29 of Han & Vardoulakis, 1991). For Ottawa sand ($D_{50} = 0.72$ mm), which was used also for the cavity inflation tests, ϕ_{res} is 28° , which was calculated from the friction angle $\phi_s = \arctan (\tau_s/\sigma_n) = 28^\circ$ along the shear band boundaries and the angle of the direction of the shear band relative to the horizontal direction $\theta_s = 56^\circ$ as reported by Drescher *et al.* (1990).

By assuming that the shear strain is uniform within a shear band, the shear strain within the shear band for Ottawa sand until the residual state was obtained as the summation of the homogeneous shear strain at the onset of strain localization = 5% (fig. 8a of Drescher *et al.*, 1990) and the $u_s^*/\text{shear band width } t_0 =$

$2.9 \text{ mm}/6.3 \text{ mm} \times 100 = 46\%$ needed to reach the residual state (fig. 12(a) of Drescher *et al.*, 1990). The parameter u_s^* is the increment of u_s between the peak state and the start of the residual state. This value is about 50%, which is very close to the value shown in Fig. 10(a). Curve 2 in Fig. 16 was obtained from equation (5) by using $\phi_{res} = 28^\circ$ ($\mu_{res} = 0.47$) and assuming that at a post-peak increment of $\gamma^p = 46\%$ the stress state almost reaches the residual state (i.e. the difference $\mu - 0.47$ is 1% of $\mu_{max} - 0.47$). It seems to us that curve 2 is more likely to be the stress–strain relation within a shear band and so we wonder why the post-peak stress–strain relations obtained from these results from biaxial tests (i.e. curve 2 in Fig. 16), rather than curve 1, were not used for the analyses.

The Paper states that although the bifurcation theory could not predict correctly the onset of strain localization observed in the tests, the deformation mode started becoming inhomogeneous at some stage after the differential cavity pressure began to drop. We think that when the post-peak branch of the curves shown in Fig. 12 is properly simulated by numerical analysis, the non-uniform stress–strain state after the onset of shear banding should be taken into account. It seems that the particle size effect could be observed in such cavity inflation experiments on sand as reported by the Authors, and then the strain localization into a shear band (or bands) having a characteristic width, which is primarily a function of particle size, should be incorporated into the analysis.

We also consider that because the strain localization was not taken into account in deriving the theoretical curve in Fig. 12, it was necessary to adjust the residual strength so that it was as low as it was for curve 1 in Fig. 16. Does this curve reflect the likely average stress versus average strain relation for a sand mass, as shown by curve 3 in Fig. 16? This curve was derived from equations (2) with $\phi_s = 28^\circ$ or $\mu_{res} = 0.47$, by assuming arbitrarily that the ratio of the shear band volume to the sand mass volume is 10%.

Curve 4 in Fig. 16 was obtained from equations (4) and (5) using the values of the parameters given in the Paper. It is different from curve 1,

Fig. 16. Post-peak stress-strain relations

although the Paper says that curve 1 was also obtained from equations (4) and (5). Which curve was actually used for the analysis?

Authors' reply

We agree that the relationship shown in Fig. 10(a) does not conform with equations (4) and (5) with constants $C_1, C_2, \dots, C_5$ as listed in the Paper. The reason for this is that equation (5) should have read as

$$\mu = C_4 \exp \{ -[C_5(\gamma^p - \gamma_p^p)]^2 \} \quad (5)$$

which is the equation that was used in the calculations.

The first paragraph on page 403 explains that '... the softening branch [equation (5)] differs significantly from that usually reported in triaxial tests; it was chosen to match the cavity inflation test results'. It was not asserted that the material true softening branch was of the form given by equation (5). However, the last argument in this quotation is weak. As the theoretical analysis assumes axisymmetric stress and strain states, it should have been terminated at the cavity volume change at which the shear bands formed in the experiments.

On a physical basis, we agree that a drop to zero shear stress is not realistic. However, the form of the softening branch only slightly affects the hardening portion of the derived curves in Fig. 12. More important is the fact that it has little effect on the theoretical prediction of the

localization at the inner or the outer radius of the cylinder (indicated in Fig. 12 by black dots). For the curve given by equations (4) and (5), point i corresponds to shear strain $\gamma^p = 0.004$ and point 0 to $\gamma^p = 0.132$, both being much smaller than values for most of the softening branch. In other words, taking a more realistic softening branch would not alter significantly the location of these points, and the discrepancy with experiments would remain unchanged.

We did not attempt to analyse the stress-strain relationship within the shear band, neither was this the objective of our earlier paper (Drescher, Vardoulakis & Han, 1990). We believe that the stresses within the shear band cannot be determined accurately from the force measurements taken at the boundaries of the specimen. As discussed by Vardoulakis, Goldscheider & Gudehus (1978), the boundaries of a shear band may coincide with stress discontinuities.

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