DISCUSSION

In situ and laboratory stress–strain relationships

M. KABBAB, F. TAVENAS and S. LEROUÉIL (1988). *Géotechnique* **38**, No. 1, 83–100

K. S. Li, Binnie & Partners, Hong Kong

The question of whether theory A or B is more representative of the in situ stress–strain relationships of clays is the subject of great debate as evidenced by discussions published (1987) on the paper by Mesri & Choi (1985a). The Authors concluded in the paper that theory A strongly underestimated the in situ strains and was therefore not valid, although Leroueil mentioned (1988) that the relative validity of theories A and B has not yet been definitely established experimentally in the laboratory. Mesri & Choi (1987) maintained the view that the computer program ILLICON developed on the EOP ϵ – σ' uniqueness concept (theory A) provided the best engineering solution for one-dimensional settlement and pore water pressure analyses and argued that the large difference between the in situ strain and that predicted using the EOP ϵ – σ' curves obtained in the laboratory is in complete contrast with common experience on computed and observed field settlements. Mesri & Choi (1987) also questioned the reliability of the in situ ϵ – σ' curves due to the large number of variables and assumptions involved in their estimation.

The results presented by the Authors did indicate a strong difference between the in situ and laboratory EOP ϵ – σ' curves. However, before any conclusion on the validity of theory A can be drawn from these observations, the following questions must be addressed.

- (a) Can the ϵ – σ' curve of presumably one sample taken at a particular level within the sublayer under study be used to represent the in situ ϵ – σ' curve?
- (b) Is the difference unexpected?

Before discussing these two questions, it is useful to consider four different types of statistically homogeneous soil profiles, all with a constant trend and the same mean value of soil property. Fig. 22 represents a soil profile which exhibits no variability of soil property (type A). For this type of profile, the in situ soil property for the entire profile can be established once the property of a single sample taken anywhere within the soil profile is known. This is at best a hypothetical soil profile. Fig. 23 shows a soil

profile with a piecewise constant variation of soil property (type B). Again, it can only be a hypothetical model. In reality, the point-to-point variation of a soil property within a soil profile is not constant. It is random and should be described as a random field. The real variation of a soil property within a soil profile is more like that of a type C or type D profile as depicted in Figs 24 or 25. A type C soil profile is one which exhibits a random but gradual variation of soil property. If the soil profile is divided into a sufficiently large number of sublayers, the variation of soil property within each sublayer is small so that it can be idealized as a hypothetical type B profile. A type D soil profile is one whose property changes so rapidly that it is not practical to subdivide the soil profile into a large enough number of sublayers to enable representation by a type B profile.

The Authors, both in the Paper and other publications (Leroueil & Kabbaj, 1987; Leroueil, 1988), although not stated explicitly, assumed that the EOP stress–strain curve of a sample at a particular level near the centre of the sublayer is representative of the stress–strain relationships of other soil elements within the same sublayer. They therefore consider that a real soil profile can be represented by a type A soil profile or at least that a soil profile subdivided into layers of thickness 1–3 m is thin enough to enable a soil profile to be idealized as a type B soil profile. The Authors' conclusion concerning the validity of theory A is based on the observation that there is a strong difference between the measured and in situ stress–strain relationships. This conclusion would be valid unconditionally if the soil profile is of type A or if the soil layer studied is a sublayer of a type B soil profile so that a single sample taken anywhere within the sublayer would suffice to represent the entire sublayer. The conclusion would be very likely to be valid if the soil profile is of type C, but questionable if the profile is of type D.

To characterize the random variation of a soil property within a soil profile, Vanmarcke (1977) defined a useful parameter called the scale of fluctuation δ which is a measure of the spatial extent within which the soil property exhibits a strong

Fig. 22. Variation of soil property for different types of soil profiles

Fig. 23. e -log σ_v' relationships for a marine deposit from Hong Kong

Fig. 24. Variability of CR and RR for Boston blue clay (after McCarron & Chen, 1987)

Fig. 25. Variability of CR for a marine deposit from Hong Kong (site 1)

statistical correlation. A large value of δ implies that the soil property is highly correlated within a large distance giving rise to a smooth variation within the soil profile (type C profile). However, a small value of δ signifies that the soil property varies rapidly (type D profile). Table A summarizes the vertical scale of fluctuation of soil properties of clays quoted in the literature or which the Writer has inferred from the data given in the stated references. It can be seen from table 1 that the vertical scale of fluctuation is small, typically less than 1 m. This implies that the real variation of a soil property tends to resemble that of a type D soil profile. A good example of this is

offered by the $e - \log \sigma'$ curves of soil samples taken at different depths from the same borehole of a marine clay deposit in Hong Kong. The $e - \log \sigma'$ of the sample at depth 4.7 m closely resembles that of the sample at depth 5.7 m, but differs drastically from that at depth 3.7 m even though they are both 1 m apart. The same is true for samples at depths of 5.7, 6.7 and 7.7 m. This suggests that samples taken 1 m apart are statistically independent and the scale of fluctuation for this clay deposit is less than 1 m.

In the case studies reported by the Authors, all the sublayers studied were no less than 1 m thick. It is believed that the stress-strain relationships

Table 1. Vertical scale of fluctuation of clays

Soil type	Soil property	δ : m	Reference
Sea bed clay	Undrained shear strength	0.17	Wu & Lee (1988)
Lierstranda quick clay	Undrained shear strength	<0.5	Lumb (1975)
Plover Cove clay	Undrained shear strength and index properties	<0.3	
Soft clays	Undrained shear strength	1.2–2.6	Matsuo (1976)
Soft Bangkok clay	Undrained shear strength	0.33	Bergado <i>et al.</i> (1987)

of the soils would vary significantly with depth and the samples taken near the centres of the sub-layers do not necessarily exhibit stress-strain relationships close to the in situ stress-strain of the soil layers.

However, there is still a condition under which the Authors' conclusion could be valid despite the fact that the scale of fluctuation is small. This is when the variability of stress-strain relationships is negligible (i.e. when the standard deviation is small) so that all types of soil profiles, A-D, are practically the same. Unfortunately, this does not seem to be true in reality. Fig. 24 shows the variation of the compression ratio CR (defined as $C_c/(1 + e_0)$) and the recompression ratio RR (defined as $C_r/(1 + e_0)$) of Boston blue clay with depth. The estimated range of $\pm 2 \times$ (standard deviation) is also shown in the Figure. Although the mean values of CR and RR both appear to be constant with depth so that the soil profile can be described as homogeneous in the statistical sense, the inherent variability CR and RR is far from being negligible. A similar conclusion can be drawn for the marine clays in Hong Kong (Figs 25 and 26). These observations suggests that the strains during loading along the virgin $e - \log \sigma'$ curve predicted using the stress-strain curves of two different soil samples can easily differ by 50% or more. This is perhaps not too significant when compared with the uncertainty arising from the variability of the preconsolidation pressure. Fig. 27 shows the measured preconsolidation pressure of the marine deposit in Chek Lap Kok, Hong

Fig. 26. Variability of CR for a marine deposit from Hong Kong (site 2)

Fig. 27. Variability of preconsolidation pressure of clay at Chek Lap Kok (after Koutsoftas *et al.*, 1987)

Kong (Koutsoftas *et al.*, 1987). The overconsolidation ratio of the soil is typically in the range of 1 to 2. As the recompression ratio RR is only a small fraction of the compression ratio CR , typically 5 to 10% (Koutsoftas, Foott & Handfelt, 1987), a small variation of σ'_p could give rise to a significant change in the predicted strain. Suppose a sublayer is loaded to twice the in situ initial vertical stress σ'_{v0} , the strain predicted using the stress-strain curve of a soil sample having an overconsolidation ratio of two would only be 5–10% of that using a soil sample which has an overconsolidation ratio of one. Although this example is extreme and may not be representative of the clays studied by the Authors, it is not impossible and does show the influence of variability of soil property on the predicted value of strain.

Conceptually, the so-called in situ strain is the spatial average strain $\bar{\epsilon}$ of the strain ϵ_z of the soil elements within the soil layer. Thus

$$\bar{\epsilon} = \frac{1}{H} \int_H \epsilon_z dz$$

The actual strain at any depth on the soil layer is not known and can only be estimated by the average strain $\bar{\epsilon}$ inferred from the knowledge on the stress-strain relationship and density of the samples taken within the soil layer. For the sake of argument, the average strain $\bar{\epsilon}$ of a normally

consolidated clay deposit may be calculated using the textbook formula as follows.

$$\bar{\epsilon} = S/H = CR \log ((\sigma_{v0} + \Delta p)/\sigma_{v0})$$

where S is the settlement, H the thickness of the layer, Δp the increase in effective stress. In general, the input values of CR , σ_{v0} and Δp are the average values of the parameters for samples obtained within the deposit, and not the parameters inferred from a particular sample. The total uncertainty in using $\bar{\epsilon}$ for predicting the in situ strain $\bar{\epsilon}$ can be broken into three components (see also Vanmarcke, 1977; Ang & Tang, 1984; Li & Lumb, 1987; Wu & Lee, 1988 and Li, 1988 for further discussions), namely

$$\begin{aligned} \text{Total uncertainty} &= \text{model uncertainty} \\ &+ \text{inherent uncertainty} \\ &+ \text{sampling uncertainty} \end{aligned}$$

Model uncertainty includes for instance the error due to the use of an imprecise theory (be it theory A or B), the simplifying assumptions involved in the calculations, the suitability of the testing procedure used to estimate the stress-strain relationships of soil samples and the like. Inherent uncertainty refers to the uncertainty arising from the variability of soil properties within the field. The third component arises from the uncertainty that the estimated mean strain $\bar{\epsilon}$ may differ from the true mean value of the in situ strain $\bar{\epsilon}$. The inherent uncertainty cannot be reduced. However, if the thickness of the soil layer is large relative to the magnitude of the vertical scale of fluctuation, the inherent uncertainty is usually a small fraction of the total uncertainty. The sampling uncertainty can be reduced by taking more samples from the field so that the average strain $\bar{\epsilon}$ can be established more precisely from the stress-strain relationships of the soil samples. For example, if the variance of a random variable is s^2 , the variance of the sample mean value will be s^2/N where N is the number of samples. In the Paper, the Authors attempted to estimate the in situ strain using the EOP stress-strain relationship of one sample and hence $N = 1$. This would give the maximum sampling uncertainty possible. In Mesri & Choi's (1985a) work, the stress-strain curves input to the program ILLICON are not the stress-strain curve of any particular sample. Instead, they are average stress-strain curves constructed presumably from the knowledge on the average values of e_0 , σ_{v0}' , σ_p' , CR , RR and so on. Therefore, the sampling uncertainty for Mesri & Choi's analysis is smaller than that of the Authors. Consequently, it is not unexpected that the predicted settlement by Mesri & Choi (1985) is closer to the in situ strains than that indicated by the Authors' although the almost exact fit of Mesri & Choi's prediction with the measured set-

tlement is surprising in view of the variability of soil properties.

In summary, the estimation of the in situ strain using the EOP stress-strain of a sample taken near the centre of the sublayer is subject to a very large uncertainty. Unless it is proven that the inherent variability and sampling uncertainty is not large enough to explain the strong difference between the in situ and EOP stress-strain relationship, the issue of whether theory A is representative of in situ behaviour remains an open question. In the Writer's opinion, the best way to establish the relative validity of theories A and B is to use remoulded soil samples prepared under identically controlled laboratory conditions. This will minimize if not eliminate the inherent sampling uncertainty associated with the Authors' approach of comparing in situ and measured EOP stress-strain curves. As argued by Mesri & Choi (1987), soil mechanics formulation intended for engineering practice almost by definition can rarely be simultaneously realistic and precise. A practical designer would be more interested in knowing whether the settlement and pore pressure predicted using theories A and B would be so large as to offset the inherent and sampling uncertainty. If not, it would not matter which theory is closer to reality.

D. J. Naylor, *University College, Swansea*

The excess pore pressure: time relationships for clay layers under the Berthierville and St. Alban embankments shown in Figs 8 and 16 respectively illustrate the surprising phenomenon of rising pore pressures after construction has ceased. The Authors note that this would not be a Mandel-Cryer effect as the geometry is essentially one dimensional and suggest that the cause is due to creep effects. They quote other cases where this phenomenon has occurred.

Another case is the 35 m high Carsington dam in England which failed close to the end of construction in June 1984. This dam had a central clay core with an unconventional 'boot' which extended the base of the core some distance under the upstream shell. Unusually high pore pressures were measured in the core during construction and particularly in two piezometers located in the boot. The point of interest here is that the pore pressure in these two piezometers continued to rise after construction had ceased in the first winter shut down period of 1982-83. In the second and final winter shut down of 1983-84 they showed a slight fall. Other piezometers in the central part of the core showed a slight rise after construction ended in 1983-84 followed by a gradual fall. The readings from the boot piezometers CC1 and CC2 are presented in Fig. 13 of Coxon (1986), reproduced here as Fig. 28.

Fig. 28. Pore pressures in fill—chainage 850 (after Coxon, 1961): pore pressure ratio expressed as r_u value, $r_u = \gamma w^h / \gamma z$

The cause of the pore pressure rise in Carsington dam is not known. The Writer has suggested that the Mandel-Cryer effect is a factor here as the situation is not one dimensional, there being differential consolidation rates between the core and adjacent shell material (*New Civil Engineer*, 1985). Creep may contribute to a Mandel-Cryer effect by increasing differential consolidation. The Authors' comments suggest that creep may have alternatively, or in addition, been a direct cause of the pore pressure rise.

This increase of pore pressure with time will, in the case of embankments, usually be associated with a reduction in safety factor. The questions of how to predict whether or not such an increase will occur and if so what will be its magnitude need to be addressed.

Authors' reply

K. S. Li and D. J. Naylor address in their discussions two interesting points, respectively the representativity of test results and the practical effects of the pore pressure increase after the end of construction of an embankment. In this reply, these two points will be considered separately.

The discussion by Li is related to the variability of natural deposits and soil properties, and questions the applicability of the approach used in the Paper to verify whether theory A or theory B is more representative. It is quite clear that for deposits presenting important variations of soil parameters, as for the Boston blue clay and Hong Kong clay deposits presented in the discussion, the method used would not be applicable. Soft

clay deposits in the Saint-Laurent lowlands and in Scandinavia are, however, generally more homogeneous and more appropriate for such studies. For the clay sublayers considered in the Paper the soil properties show very little variability and vary with depth in a monotonous manner. This is the case for the preconsolidation pressure, as shown in Figs 2 and 13 for Berthierville, Fig. 14 for Saint-Alban and Fig. 20 (also Chang 1981) for Väsby. In such circumstances, and clearly only for the 90 cm thick sublayers at Berthierville, it can be considered that the stress-strain behaviour measured on specimens taken in the central part of the sublayer is representative of the sublayer and can be compared with the observed in situ stress-strain behaviour. Only MSL_p and MSL_{24} stress-strain curves are shown in the Paper but it must be said that the stress-strain behaviour of the clay in the central part of the sublayers has not been characterized by these two tests only but by a series of oedometer tests (18 for the 3.0–3.9 m sublayer and 14 for the 3.9–4.8 m sublayer at Berthierville; 18 for the 3.1–4.9 m sublayer at Saint-Alban). Figs 29 and 30, from Leroueil, Kabbaj & Tavenas (1988) present the results of these tests in the form of $\sigma_v' - \epsilon_v$ relationships for strains ϵ_v of 2.5, 5, 7.5 and 10%. The scatter, or the variability, is very small for the clays considered and the MSL_p and MSL_{24} test results are well in the range defined by the other tests.

The figures also confirms that the clay behaviour is controlled by an effective stress-strain rate relationship ($\sigma_v' - \epsilon_v - \dot{\epsilon}_v$) as demonstrated by

Fig. 29. Effective stress-strain rate variations at different strains for Berthierville clay (from Leroueil *et al.*, 1988)

Leroueil *et al.*, 1985). In situ conditions obtained at the various strains are plotted on the figures; at least for strains greater than 5%, they are in very good agreement with the $\sigma'_v - \varepsilon_v - \dot{\varepsilon}_v$ model established in laboratory. It is logical that MSL_p stress-strain (EOP) curves involving strain rates much larger than in situ strain rates give strains smaller than in situ at a given effective stress (note that due to an error in the drawing process the symbols for the MSL_p and in situ stress-strain curves are reversed on Fig. 12 of the Paper).

As for the phenomenon of rising pore pressures discussed by Naylor, it appears to be quite common under embankments and can be, in general, the result of numerous factors: Mandel-Cryer effect, creep, progressive failure (Tavenas & Leroueil, 1980), problems with instrumentation. For the Berthierville embankment creep effects seem to be the only reasonable explanation. For the Carsington dam, the situation is quite differ-

ent because the conditions are far from one dimensional and the material involved is not a soft clay.

Even in a simple case as at Berthierville, the magnitude of the increase in pore pressure after the end of construction depends on many factors: compressibility and creep characteristics, geometry of the problem and in particular the thickness of the clay layer, and the permeability of the clay. At the present time, there is no reliable method to predict pore pressure increase. Whatever one's ability to predict them, as indicated by the Writer, such pore pressure increases must have a detrimental effect on the stability. However, to the Authors' knowledge, no delayed failure associated to pore pressure increase after the end of construction has been reported. One possible explanation could be that while there is pore pressure increase in some parts of the foundation, there is a decrease at some other places and globally the embankment remains stable.

Fig. 30. Effective stress-strain rate variations at different strains for the Saint-Alban clay (from Leroueil *et al.*, 1988)

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