ship for long slender piles. As such, it gives a good fit to the curves for the longer piles in Fig. 4 of the Paper.

For homogeneous conditions where the skin friction τ_s is independent of depth, equation (5) becomes

$$\frac{w_t - w^*}{d} \approx \left[\frac{P_t}{P_{ult}} \right]^2 \frac{\pi \tau_s l^2}{2 E_p A_p} \quad (7)$$

and the expression for the reduction factor λ_r at $w_t = 0.1d$ is

$$\lambda_r = \left[\frac{2}{\pi} \left(0.1 - \frac{w^*}{d} \right) \frac{E_p A_p}{\tau_s l^2} \right]^{1/2} \quad (8)$$

The current trend for using larger diameter piles to support offshore structures means that the calculated reduction ratio λ_r will rarely be much less than unity. For example, a 100 m long, 1.5 m dia. pipe pile, with wall thickness of 50 mm would need to be in soil with $k \geq 4$ kPa/m (implying a β value close to 0.4), which is high for a soft clay deposit. Alternatively, for a stiffer, more homogeneous soil deposit, τ_s would need to be greater than 250 kPa for λ_r to be less than unity. However, the present data base for long piles contains pile tests performed on very compressible, long slender piles, where λ_r will be considerably less than unity. Such load tests have often been performed soon after pile installation, before excess pore pressures have fully dissipated. There is clearly a major need for more reliable test data on long piles where the appropriate stiffness ratios match those for prototype offshore piles.

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Author's reply

Dr Randolph's elegant, yet simple, analysis demonstrates the value of having closed-form solutions to engineering problems. Such solutions not only provide a check on corresponding solutions from numerical analyses, but also help to identify the dimensionless parameters which control behaviour. Hence, Dr Randolph has found that greater generality is achieved if one uses a stiffness ratio defined in terms of the rate of increase of skin friction k , rather than the rate of increase of soil modulus N_v . The use of the latter definition is indeed appropriate when considering settlements in the elastic (working) range and can also be advantageous when cyclic degradation of skin friction is being incorporated into the analysis (Poulos, 1981).

There is little doubt that, although pile compressibility contributes to the length effect on skin friction, there are other factors which may also be significant, particularly strain-softening at the pile-soil interface. For example, model tests by Chandler & Martins (1982) indicate that reductions of up to 50% in the skin friction can occur following relatively large displacements. This effect would result in smaller values of the reduction rates λ_r than are indicated by the present analysis, and an analysis incorporating this strain-softening behaviour would give better overall agreement with the available data.

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Cone penetrometer and liquid limit

D. M. WOOD (1982). *Géotechnique* **32**, No. 2, 152–157

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The Author provides further evidence as to the usefulness of index testing by means of cone tests, as does Houlsby (1982). However, in

reviewing past data on estimates for shear strengths at the liquid limit, allowance has to be made for variations in test methods, particularly with regard to the base hardness of percussion cup devices (Casagrande, 1958). Norman (1958) showed that the BS 1377 (1975) test produces higher liquid limits than does the Casagrande test and that the shear strengths estimated for the two

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Fig. 1. Relation between shear strength and liquidity index (after Skempton & Northey, 1952)

tests can differ, possibly by 30–60%. Youssef, El Ramli & El Demery (1965) used a Casagrande device for their tests, and thus Fig. 1 and Table 1 in the Note may not correlate with BS 1377 results for liquid limit. Karlsson (1977) specifically excluded BS 1377 equipment from his investigations because of the difference between this and the liquid limit tests used in Sweden. The BS fall cone test may now produce results which correlate more favourably with the Swedish cone test and fortuitously result in a reasonably consistent international standard for the liquid limit index test (Whyte, 1982).

The Author shows that the assumption of unique strengths at both liquid and plastic limits provides a valuable framework for comparing site investigation data, subject to recognized caveats. However, one he does not report is the necessity to allow for the effect of coarse material on the moisture content of in situ soils. In preparing for index tests, it is necessary to sieve out material coarser than 425 μm , although in site investigations the moisture content of the whole sample is normally reported. Suggestions as to how this may be allowed for are given by Forde (1978) and Arrowsmith (1978).

The Author assumes a factor of 100 between the strengths of soils at their plastic and liquid limits,

Fig. 2. Relation between liquidity index (consistency index) and shear strength of remoulded clays (after Skempton & Northey, 1952)

thereby taking values of 170 kN/m^2 and 1.7 kN/m^2 respectively. The latter value is accepted as being of the correct order of magnitude, but the value for the plastic limit could tend to be too high to provide an approximate correlation with the results of rolling bead tests. Fig. 1 shows the Skempton & Northey (1952) data which apparently imply a factor of 100 over the plastic range. However, British Standard methods were used to determine liquid limit and relate to strengths of the order of 1.0 kN/m^2 , whereas the 1.7 kN/m^2 reported here refers to cone tests and the harder base Casagrande tests of Youssef *et al.* (1965). If a value of 110 kN/m^2 is assumed at the plastic limit (see Fig. 1), this agrees with averages from rolling bead tests (Whyte, 1982) and also represents a consistency state about mid-way in the stiff range for soils. Fig. 2 shows the data from Skempton & Northey (1952) replotted on the basis of 1.6 kN/m^2 and 110 kN/m^2 for strengths at the index limits of plasticity, and the apparent empirical uniqueness of strength with liquidity has been confirmed with tests on a range of British soils. Thus the assumed strengths over the plastic range can be estimated to be about a factor of 70 (from 1.6 to 110 kN/m^2) for cone liquid limits and plastic limits, with the caveat that plastic limits are notoriously unreliable and require re-examination to develop a satisfactory reproducible index test for soils. The Author reports Lawrence (1980) as finding a ratio of less than 100 over the plastic range for speswhite kaolin, but does not report the observed factor. Could this be nearer the value of 70 suggested here?

I agree that the water content at which a 30°,

80 gf cone penetrates 20 mm is a useful index property which appears to relate to a strength of about 1.6 kN/m^2 . This would tend, on average, to produce lower liquid limits than would BS 1377 percussion cup tests. Similarly, the water content at which saturated remoulded samples possess a strength of 110 kN/m^2 is a useful index property which appears to relate to plastic limit values. The range of soil strengths between these index values is a factor of about 70.

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The Author commends the cone penetrometer liquid limit as a useful index property for soils and suggests that it is natural to attempt to use the cone penetrometer to determine the plastic limit. He mentions that if the liquid and plastic limits are assumed to correspond to unique strengths for all soils, then a simple relation exists between remoulded shear strength and liquidity index (Schofield & Wroth, 1968), but he is aware that there are possible difficulties if such a relation is used to derive the plasticity index and hence the plastic limit. Despite this, he suggests that a plasticity index so derived, which will indicate the change in water content which will correspond to a one hundredfold change in shear strength, may be

more useful to the engineer than is the plasticity index derived from the traditionally determined liquid and plastic limits.

There is clearly a requirement for a method of measuring the plastic limit which not only offers improved repeatability and reproducibility in comparison with the plastic limit determined by the Casagrande method, but which also gives a result which relates more directly to soil behaviour in the field. A method developed at the Scottish Institute of Agricultural Engineering (Campbell, 1976) which has met with approval from agricultural soil scientists and agricultural engineers uses a drop-cone penetrometer.

The method uses the same apparatus and soil preparation as the BS cone penetrometer liquid limit test. Drop-cone penetration is measured in about ten soil samples with water contents in the region of the plastic limit. When such water content/cone penetration results are plotted a pronounced minimum penetration is always observed. The water content corresponding to this minimum defines the cone penetrometer plastic limit and it is found to correlate closely with, but is not the equivalent of, the Casagrande plastic limit, the latter always being greater by about 2–15% w/w.

Initially, an attempt was made to define the cone penetrometer plastic limit in terms of penetration rather than water content because different soils would then have similar strengths at their plastic limits, but the presence of a minimum on the water content/cone penetration curve implies that any given penetration may be attainable at two water contents or not at all. Thus, the cone penetrometer plastic limit was defined as the water content corresponding to the minimum of the curve. However, the penetrations corresponding to the minimum water contents for 16 agricultural topsoils, one subsoil and the clay mineral kaolin were found to have an average of 1.36 mm, a range of 0.4–1.8 mm and a coefficient of variation of only 30%.

It follows that the method has an attraction in that different soils have similar strengths at their cone penetrometer plastic limits. In addition, the test is no longer subjective and offers reproducibility at least as good as the Casagrande test, even when carried out by inexperienced operators. Moreover, as it is usually the plasticity index rather than the plastic limit *per se* that is of interest for soil classification and the prediction of soil behaviour, it is also advantageous to measure both the liquid and plastic limits simultaneously simply by carrying out the drop-cone test over a suitable range of water contents. Such a procedure allows the plasticity index to be determined in half the time required by the Casagrande methods, the

principal gain being in the liquid limit determination. A further advantage of using the method is that, unlike the Casagrande plastic limit, all soils have a cone penetrometer plastic limit. This accords with both the critical state theory of soil mechanics and the common experience that, given appropriate confining stresses and water content, any soil can undergo plastic deformation irrespective of whether or not it is non-plastic according to the Casagrande plastic limit test.

Campbell (1976) suggested that the minimum on the water content/cone penetration curve implied a change in the physical state of the soil and that the water content corresponding to the minimum is a better indicator of the water content at which soil behaviour changes from brittle to plastic than is the Casagrande plastic limit which is more empirical in origin. In order to test whether or not the cone penetrometer plastic limit does correspond to a change in soil behaviour, Campbell, Stafford & Blackwell (1980) examined the variation with water content of several agriculturally important soil properties including cohesion, soil-metal friction, susceptibility to compaction, agricultural implement draught and the slope and intercept of the virgin compression line of the critical state theory of soil mechanics. For a given soil, all these relations exhibited turning points at a similar water content which corresponded to the cone penetrometer rather than the Casagrande plastic limit. It was concluded that the cone penetrometer plastic limit was the better indicator of soil behaviour in the field and of the water content at which a soil changes from the brittle to the plastic state.

What are the Author's views on the relevance to civil engineers of this work?

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Author's reply

Both of the discussion contributions are concerned with the relations between index properties and other useful mechanical properties of soils—a wider subject than that dealt with in the Note.

The experimental observation of critical state lines for a number of soils shows that over a moderate range of stress a linear relationship between water content and logarithm of undrained strength can be produced for each soil. The

strengths that are appropriate to critical state conditions are strictly remoulded strengths: at the critical state the soil is being continuously churned up and any effect of structure has been destroyed.

If a particular soil has a strength at a water content equal to the liquid limit C_{LL} and if the ratio of the strengths of that soil at water contents equal to the plastic limit and liquid limit is R then the linear relationship between water content and logarithm of strength C_u can be written in terms of liquidity index LI as

$$C_u = C_{LL} R^{1-LI}$$

or

$$LI = 1 - \frac{\log C_u / C_{LL}}{\log R}$$

This sort of relationship is useful particularly when related soils—of similar clay mineralogy, from geographically similar sites—are being considered. For these a single pair of values of C_{LL} and of R may reasonably be supposed to obtain, and a consistent pattern in data of C_u and LI can be sought in a plot of LI against $\log C_u$ without worrying about the actual numbers.

However, as an aid to estimating values of remoulded strength for an almost unknown site where only index property and water content data are available, bold assumptions must be made about actual values of C_{LL} and R . The data shown by Skempton & Northey (1953) relate only to four soils. Mitchell (1976) shows data for many more soils and it seems that if round numbers are shown for convenience then $R = 100$ and $C_{LL} = 1.7 \text{ kN/m}^2$ is not too bad a first shot. It could be reasonably argued that 2 kN/m^2 would be a rounder number to assume for C_{LL} —and this would not change dramatically the estimates of remoulded strength that might be made.

Undisturbed soils will have a structure which can be measured, to some extent, by their sensitivity. Typically the compressibility of undisturbed soil is greater than the compressibility of the same soil when remoulded and the estimates of compression index plotted in Fig. 6 of the Note using the expression $C_c' = PI \times G_s / \log_{10} R$ (with $R = 100$) could be expected to err on the low side. Typically, too, the sensitivity of undisturbed soils of a given clay type increases with liquidity index from about unity at the plastic limit ($LI = 0$). Incorporation of this liquidity-dependent sensitivity will reduce the apparent value of R when undisturbed strengths are plotted against liquidity index. Again, however, a plot of values of C_c and PI for related soils should show a consistent pattern irrespective of the appropriate value of R .

Whyte proposes a value of $R = 70$ and introduces a value for $C_{LL} = 1.6 \text{ kN/m}^2$. Lawrence

(1980) suggested a value of $R = 25$, but this seems to be an anomalous value specific to the speswhite kaolin that he was testing. Plots of water content against logarithm of shear strength are typically curved, showing a more rapid increase of $\log C_u$ with decrease in w as water content decreases. A value of R deduced from the shape of this relation near the liquid limit may tend to underestimate the average value of R that might apply between the liquid and plastic limits.

The discussion in the Note about the relation between strengths and cone penetrations revolved around the selection of a cone factor K for the expression

$$C_u = KW/d^2$$

A value of $K = 1.2$ for 30° cones was taken from Hansbo's (1957) work. Houlsby's (1982) analyses suggest that a value of K (the reciprocal of his F) of 0.96 might be appropriate for a perfectly smooth cone, or 0.51 for a perfectly rough cone. Gardiner (1982) used a miniature remoulding vane to estimate shear strengths and suggests that a value of K of about 1.0 may be reasonable.

With $K = 1.0$ a cone penetration of 20 mm corresponds to an undrained strength of 1.96 kN/m^2 —which might be a further argument for taking C_{LL} as 2 kN/m^2 for liquid limits determined with the cone penetrometer following the procedure laid down in BS 1377 (1975). A strength of 1.7 kN/m^2 corresponds now to a cone penetration of 21.5 mm, and a strength of 1.6 kN/m^2 to a penetration of 22.1 mm. With a new value for K the relative disposition of the data of Youssef *et al.* and of Sherwood & Ryley in Figs 4 and 5 of the Note will be slightly changed.

I am grateful to Campbell for drawing other work to my attention. He describes a test which can be carried out using the cone penetrometer to determine a plastic limit for soils as the water content at which the penetration of a 30° , 80 gf cone reaches a minimum. It would be interesting to find an explanation for the existence of this minimum and to discover whether or not there is a combination of factors which controls the penetration of the cone at low water contents and which might serve to account for the apparent range of about 20 ($1.8^2/0.4^2$) in strengths of soils at the cone penetrometer plastic limit. The ratio of strengths at

the cone penetrometer plastic limit and at the liquid limit has an apparent range of 2500 to 123 with an average of 216. It is not clear how the cone penetrometer plastic limit gives an indication of the water content at which a soil changes from the brittle to the plastic state.

The suggested saving in time for the determination of the plastic limit using this technique seems questionable. An investigation of cone penetrations at a range of low water contents must be a slower process than rolling out a thread of clay until it crumbles. One might expect there to be problems of mixing sufficient clay to fill the cone penetrometer pot to a uniform water content, but at the penetrations mentioned the volume of clay participating in the penetration will extend only a few millimetres around the tip of the cone and problems of heterogeneity will probably not be significant.

I suggested in the Note that the water content change for a one hundredfold change in strength was more useful to the civil engineer than the plasticity index derived from liquid and plastic limits. Consequently I was not interested in discovering tests which might lead to values of plastic limit but rather in a rational interpretation of cone tests which permits the compressibility of soils—and hence the relation between strength and water content—to be deduced. Whereas I was perhaps seeking to abandon the plastic limit, Campbell is striving to preserve it.

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