calculation of settlements of larger foundations and this is demonstrated by an analytical study (Brown & Gibson, 1979). This shows that the settlements are less than for the elastic half-space, by an amount that increases with the size of the foundation and with the rate of increase of stiffness with depth.

The modulus used in such calculations should be appropriate for the strain and time involved. The dependence on these variables is being studied and I believe that there may be a satisfactory mathematical model that will relate moduli

measured under different conditions. Another dynamic method of measurements will be helpful.

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A one point cone penetrometer liquid limit test?

C. R. I. CLAYTON and A. W. JUKES (1978). *Géotechnique* **28**, No. 4, 469–472

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The test factors obtained by the Authors, and quoted in Table 1 for all soil, are virtually identical with figures obtained in South Yorkshire in 1977 even though the South Yorkshire figures were based on a sample size less than one fifth of that of the Authors and no doubt on different soils. It was also found that the reproducibility and accuracy of results using a one point cone penetrometer assessment were superior to those obtained by the one

point Casagrande method. Of the liquid limits determined by the one point cone penetrometer test 94% fell within $\pm 2\%$ of the four point test result. In view of the work that has now been carried out and the substantial cost benefits, I feel it would be appropriate for the British Standards Institution to consider the adoption of a one point cone penetrometer test and to investigate the possible use of the cone penetrometer in the determination of the plastic limit.

A natural compression law for soils (an advance on $e-\log p'$)

R. BUTTERFIELD (1979). *Géotechnique* **29**, No. 4, 469–480

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The Author expands elegantly on the traditional linear $e-\log p'$ curves. Using natural strain rather than conventional strain straightens out non-linearities often found at great strains, without greatly affecting stress-strain relations for small strains. The simplicity of the Author's concept is attractive, but the implication that it may be applied to a variety of soils, including perhaps granular soils, and to rebound and reload curves, requires further examination.

The example plots appear to show good straight-line relations for rebound and reload curves, although they are only slightly better than on standard plots. However, there are many

examples of clay soils (e.g. Hansen & Mise, 1964) showing rebound and reload curves that can be fitted only to far more complex mathematical expressions.

The relationship $d(\ln v) = -C d(\ln p')$ works only when $m_v = C/p'$. For granular soils, this relationship does not work; here, another more general approach must be taken, such as that described by Janbu (1963). Janbu proposed a soil compression modulus that depends on stress level

$$K = 1/m_v = dp/d\varepsilon = cp^{1-s} \quad (1)$$

For $s = 1$, this is equivalent to Hooke's law; for $s = 0$, it leads to

$$dp/p = cd\varepsilon = -c dv/v \quad (2)$$

Fig. 1. Compressibility characteristics of sands in CR stress paths: (a) dense River Welland sand $CR = 1$; (b) dense River Welland sand $CR = 1/K_0$; (c) dense River Welland sand $CR = 4.7$; (d) loose River Welland sand $CR = 1$; (e) dense glass ballotini $CR = 1$

Taking ε as the natural strain, this is precisely the result obtained in the Paper.

For silts and sands, however, s varies between zero and unity, and is often close to 0.5, giving

$$p^{s-1} dp = c d\varepsilon = -c dv/v \quad (3)$$

or

$$\varepsilon = \frac{1}{sc} (p^s - p_c^s) = \ln \frac{v_0}{v} \quad (4)$$

with p_0, v_0 as strain origin.

This relationship will not appear as a straight line on a plot of $\log(v_0/v) - \log p$, but will be straight on a $\log \varepsilon - \log p$ plot. In practice, good straight lines are often obtained only if minor initial strain adjustments are made to compensate for seating errors and imperfect virginity of the soil (Schmidt, 1967).

Rebound and reload curves for granular soils (and hence for preloaded sands) are substantially more complex, but can yet be normalized (Steen, 1967).

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From a study of the more compressible soils comprising clays, silts and peats the Author has shown that these soils on loading and unloading give clearly more realistic linear relationships between $\ln v - \ln p'$ (where $v = (1 + e)$ is the specific volume) than with the conventional $e - \ln p'$ plot. The improved linearity would therefore simplify the assignment of representative values for the compression and swelling indices C_c and C_s respectively (equivalent to C_c and C_s from the $e - \log_{10} p'$ plot).

From the Author's Fig. 3 I note that an added bonus, particularly to the practising engineer, is that the preconsolidation pressure p'_c is clearly indicated as a point of discontinuity in slope and therefore does not require the use of tangent fitting for its determination.

Use of the improved linear relationship to derive

Fig. 2. $\varepsilon - \ln p'$ and $\varepsilon - p'^{0.35}$ relationships for dense River Welland sand

a simple non-linear theory of virgin consolidation and revised expressions for the virgin and critical state lines in the Cam clay model makes this Technical Note commendable. However, the Author does not give data for sands.

Some compressibility data I obtained at the University of Manchester for fine subangular River Welland and spherical glass ballotini sands in constant stress ratio (CR) axisymmetric compression tests for a loading and unloading cycle are plotted in Fig. 1. The dense River Welland sand data range from the isotropic consolidation ($CR = 1$) stress path to the $CR = 1/K_0$ stress path and to a value of $CR = 4.7$ where the sand is strongly dilatant on first loading. The peak effective stress ratio R_{peak} is approximately 5.6 in axisymmetric compression for this material in the low pressure range on freshly prepared samples. Comparative data are provided by the loose River Welland sand and dense glass ballotini results.

In all cases the $\ln v - \ln p'$ plots for discrete sand particles remain non-linear, clearly unlike the behaviour of more compressible finer grain soils of clays and silts and organic material.

Fig. 3. $\ln \varepsilon - \ln p'$ relationships for subangular River Welland sand

Only the isotropic consolidation stress path is now considered.

In Fig. 2 the natural volumetric strain $\mathbb{E}$ obtained for dense River Welland sand on unloading (swelling) is plotted with a non-linear (Hertzian type) relationship p'^m and compared directly with the $\ln p'$ plot using an arithmetical scale. The value of the exponent m is 0.35 for this sand and state of packing and its evaluation is considered in Fig. 3. A linear relationship exists between $\mathbb{E} - p'^{0.35}$, whereas in keeping with Fig. 1, a non-linear relationship is obtained between $\mathbb{E} - \ln p'$.

Therefore, compared with the Author's proposed relationship for more compressible soils given in his equation (6) as

$$\begin{aligned}\mathbb{E} &= - \int_{v_0}^v (dv/v) = - \ln(v/v_0) \\ &= \int_{p'_0}^{p'} \mathbb{C} d(\ln p') = \mathbb{C} \ln(p'/p'_0)\end{aligned}\quad (1)$$

the relationship applicable to sand is

$$\begin{aligned}\mathbb{E} &= - \int_{v_0}^v (dv/v) = - \ln(v/v_0) \\ &= \int_{p'_0}^{p'} k d(p'^m) = k(p'^m - p'_0{}^m)\end{aligned}\quad (2)$$

where the Hertzian contact type constants k , the compressibility factor and m are dependent on the composition, shape, size and packing of the sand and differ for corresponding loading and unloading strains (El-Sohby, 1964, 1969).

As the value of the initial consolidation stress p'_0 can be set experimentally quite small in isotropic consolidation tests, then at higher values of p'

$$p'^m - p'_0{}^m \rightarrow p'^m$$

and a first order approximation to the values of m and k can be simply determined from

$$\mathbb{E} \simeq k p'^m \quad (3)$$

Equation (3) becomes exact when absolute volumetric strains are used from the imaginary zero datum stress ($p'_0 = 0$) and these can be obtained for a given sand and state of packing from the CR elastic model referred to below.

By taking natural logarithms on each side of equation (3)

$$\ln \mathbb{E} = m \ln p' + \ln k \quad (4)$$

A $\ln \mathbb{E} - \ln p'$ plot should therefore be linear (cf the linearity of $\mathbb{E} - \ln p'$ for more compressible soils shown by the Author) with slope equal to m and by extrapolation the value of k can be determined from the intercept on the ordinate axis (using arithmetical scales). Values of m and k obtained for

dense and loose River Welland sand using absolute strains during unloading are shown in Fig. 3 and the linear nature of the plots substantiates the use of the Hertzian type relationship given in equations (2) and (3).

The Author has indicated how the new compression law given by equation (1) gives revised equations for the Cam clay soil model. For sand use of equations (2) and (3) with unloading (swelling) m and k parameters would give the recoverable volumetric strains. A model based on the experimental unloading data (corrected for apparatus effects) from a range of CR stress paths was proposed by McDermott (1970) which enabled a prediction to be made of the recoverable (elastic) strains in any axisymmetric stress path. From the model the magnitude of the elastic strain components in the axial and radial directions, ε_1^e and ε_3^e respectively, can be determined and from

$$\varepsilon_{1,3}^T = \varepsilon_{1,3}^e + \varepsilon_{1,3}^s \quad (5)$$

where subscripts 1 and 3 refer to the strains in the axial and radial directions respectively, the corresponding non-recoverable (slip) strain components ε_1^s and ε_3^s can be isolated from the measured total strain data ε_1^T and ε_3^T .

The use of slip strains is strictly required in Rowe's (1962) stress-dilatancy relationship because this is based on slip due to rigid body sliding. Noticeable improved agreement was obtained during pre-peak loading in all drained axisymmetric compression stress paths using the elastic model to isolate the slip strain data. Details of the CR elastic model for sand are given by Rowe (1972) and McDermott (1972, 1980).

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The Author has obtained exactly the same compression law for soils as I did some years ago (Juarez-Badillo, 1965). He adopts the symbols $\mathbb{C}_c$ and $\mathbb{C}_s$ where I used γ and γ_p respectively.

I started to work on this subject in 1960 and soon had comprehensive experimental data on Weald clay, obtained at London, kindly made available by D. J. Henkel. A summary of my theories was published subsequently (Juarez-Badillo, 1975; see also Bibliography).

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Author's reply

Dr Schmidt and Dr McDermott refer to compressibility measurements on sands for which the pv^F law does not apparently provide a good general interpretation; nor does the e – $\log p$ relationship. (Dr McDermott's Figs 1(b) and 1(c) tests are for stress ratios different from unity and therefore include volumetric strains due to dilatancy.) They both suggest essentially identical compression laws, initially due to Janbu (1963), of the form

$$d\mathbb{E} = -dv/v = C dp/p^{1-s} \quad (1)$$

p being effective mean stress, which reduce to my equation for $s = 0$ and linear elasticity for $s = 1$.

Integrating equation (1) leads to Dr Schmidt's equation (4)

$$\left. \begin{aligned} \mathbb{E} &= -\ln(v/v_0) = (c/s)(p^s - p_0^s), \quad s \neq 0 \\ \mathbb{E} &= -\ln(v/v_0) = c \ln(p/p_0), \quad s = 0 \end{aligned} \right\} \quad (2)$$

for a strain zero at (p_0, v_0) which is also equivalent to Dr McDermott's equation (2). In all cases other than $s = 0$ or 1, both c and s will have very complicated dimensions, different for every value of s which is a particularly unsatisfactory feature of such power laws, whereas for $s = 0$, $c = \mathbb{C}$ is dimensionless.

Universal applicability of the pv^F law was not really anticipated although an advance on e – $\log p$ was claimed due to the way in which natural volumetric strains are incorporated, and a non-linear (small strain) extension of the Terzaghi consolidation equation can be developed, together with two practically useful results indicated by Fig. 5 and equation (24) in the Technical Note.

I was not previously aware of Professor Juarez-Badillo's (1965) work. In his paper he postulates an isotropic compression and swelling law of the form

$$dv/v = -\gamma d\sigma/\sigma$$

and therefore

$$v = v_0(\sigma/\sigma_0)^{-\gamma} \quad (3)$$

(v, v_0) being sample volumes and (σ, σ_0) corresponding isotropic consolidation pressures. He showed that this equation can provide a good fit to Henkel's (1959) data on Weald clay. The paper summarizes earlier work and, following a statement of equation (3), embarks on the prediction of volumetric strains in both normally consolidated and overconsolidated clays in triaxial compression and extension.

Professor Juarez-Badillo makes no mention of specific volume, logarithmic plots, natural strain (or stress), m_v – p' relationships, the 'natural' non-linear consolidation equation, stress-ratio effects or the concomitant modification of the Cam clay equations, so the overlap of his paper with my work seems to be confined to equation (3).

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