(1968), together with the analogue and theoretical results shown in Table 1. The shape factors at a depth B below the impervious floor may be assumed to approximate to the values given in the tables of Youngs (1968) for cavities at depth and at heights B above an impermeable floor, since even at the extreme values of $B = 0$ for $L/D > 0.5$, the shape factors obtained in this way are in error by only 20% even though the flow is through the cylindrical sides only because of the impermeable floor. Thus for values of B/D not equal to zero or infinity, for which we have accurate values obtained by electric analogue, we may confidently use Youngs' analogue results using interpolated values for low values of B/D . Table 2 may thus be regarded as a comprehensive table of shape factors to be used in the intake formula (equation (1)).

Author's Reply

I would like to thank Mr Youngs for his most interesting contribution. The historical introduction is particularly elegant, laying emphasis as it does on the fact that the originators of the various theories were only too well aware of their approximate nature. His Table 2 should be a most valuable aid in obtaining more satisfactory permeability values. Though inspection of it shows how very rapidly the shape factor varies at small values of L/D . Consequently the depth of the borehole and lining tubes must be known with adequate precision if reasonable accuracy is to be obtained.

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A discrete numerical model for granular assemblies

CUNDALL, P. A. & STRACK, O. D. L. (1979). *Géotechnique* **29**, No. 1, 47–65.

B. C. Burman, Coffey & Partners Pty Ltd

The Authors have produced an interesting paper on the application of numerical modelling of discontinua to the investigation of constitutive relationships for granular materials. The Writer (Burman, 1972) was involved in similar work between 1969 and 1971, the results of which are belatedly published (Trollope & Burman, 1980).

The senior Author and the Writer have independently developed similar but different approaches to the numerical analysis of discontinua with a view to mathematically modelling the behaviour of jointed masses in problems of rock mechanics. While Cundall chose to

Fig. 1. (a) Numerical model with load distribution for rigid base condition; (b) numerical approximation and physical model for 30° wedge of sorted cylindrical blocks

employ a dynamic finite difference-based technique and the Writer developed a static finite element-based technique, the central common feature of these developments has been the assumption of negligibly small deformability of the individual units of the discontinuum. This assumption results in a considerable reduction in the scale of the numerical problems (by at least a factor of 3) and allows the essential mechanics of discontinua to be more readily appreciated. While there are undoubtedly advantages associated with either technique, both parties seek to solve basically similar problems. The Writer is therefore inclined to the simple view that, in spite of differences in detail, similar computational overheads are involved in solving the similar problems. The different approaches are as much an expression of individual backgrounds and training as they are of relative computing efficiencies. Any further discussion of the particular merits of the technique is incidental to the main topic of the behaviour of idealized

granular masses. It is preferable to concentrate discussion on the aspects of model verification and use of numerical modelling in studying constitutive relations for granular masses.

The Authors' approach to the important question of model verification is convenient but unconvincing. As they note, the results of their verification test are 'primarily qualitative and therefore subjective'. Verification is difficult for discontinuum modelling and the Writer knows of no simple or complete answer. There is certainly a most useful area for further research in this topic. The Writer's approach was to test an instrumented physical model consisting of steel cylinders formed into a half wedge configuration with a deformable base (Fig. 1). However, due to the significant effects of dimensional tolerance in a regular granular array, the Writer's results were also in part qualitative. Indeed, it may be that verification will remain circumstantial until a major effort perhaps along the lines of photoelastic modelling pioneered by Chappell (1972) is undertaken specifically to verify discontinuum numerical models.

Although the Paper deals essentially with the mathematical derivation of the model and peripherally with verification, it suggests that the technique has application in investigating the mechanics of granular assemblies.

The Writer has carried out a study of the behaviour of idealized granular masses through an extensive series of numerical experiments involving a regular hexagonal packing of uniform cylinders arranged into a wedge configuration (Fig. 3 of Trollope & Burman, 1980). The wedge was subject to simulated gravity loading with a variety of subsequent basal deformations so that other than homogenous conditions could be evaluated. These tests lead the Writer to the conclusion that numerical experimentation with regular uniform assemblies of particles is a logical starting point in studying the mechanics of real granular materials. This belief is based on recognition of the necessity for understanding the respective roles of the various deformational mechanisms available to discontinuous systems before attempting to investigate the more general but more complex question of irregular, non-uniform assemblies. In addition to the conceptual advantage associated with regular assemblies, there is also a real practical advantage in being able to determine continuous stress and strain fields throughout the assemblage from the results of numerical experiments. This advantage derives from the ease with which repeatable units suitable for determining averaged stress and strain values can be defined. Figure 4

Fig. 2. Contour plot of actual deviation angle, i , between stress and strain rate axes for convex base deflexion of 30° numerical wedge with shearless contacts

of Trollope & Burman (1980) shows the repeatable units which were used to determine normal and shear stresses for the regular hexagonal array. Stress fields thus determined were shown to satisfy equilibrium conditions while strain fields similarly determined were shown to satisfy continuity requirements. The definition of consistent stress and strain fields for irregular assemblies would be much less easily established.

An advantage which arises from the ability to define continuous stress and strain fields through an idealized granular mass is that equivalent continuum based deformational parameters can be derived throughout the mass. The Writer, for instance, was able to investigate the developing relationship between stress, strain and rate vectors as the idealized mass was progressively deformed. Details are presented by Burman (1971) but it is worthwhile noting a result predicted analytically by de Josselin de Jong (1971) in respect of the double-sliding, free-rotating model. Results from the Writer's numerical experiments (Fig. 2) show an independence of the stress and strain rate vectors with increasing distortion of the mass that corresponds to a non-coaxial plasticity model in which the angle of divergence is a function of distortion through the mass. The Writer would be interested in investigation of these analytic and numeric predictions for idealized granular arrays by physical experiments with real granular materials. This proposition is an example of the potential benefits available to geomechanics from the development and application of numerical models of idealized discontinua.

The Writer appreciates the Authors' efforts to understand the behaviour of granular material through the application of what are often thought to be over-complicated numerical techniques. This field seems to offer considerable potential.

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Professor M. E. Harr and Professor R. D. Holtz, School of Civil Engineering, Purdue University, USA

The distinct element method and its use to describe the mechanical behaviour of an assembly of discs are indeed intriguing developments. The Authors are to be congratulated for the innovative use of interactive computer graphics. Can the method realistically describe the behaviour of real collections of particles?

Contrary to popular opinion, in a transverse section through a soil mass, even for 'homogeneous soils', few particles will be in contact with each other in that plane. To illustrate this, consider the idealized packing of perfect spheres (only 4 shown) in Fig. 1 (which is similar to Fig. 5 in the Paper). The trace of the plane AA' will exhibit the same circular array as shown in the figure. However, all other planes, such as BB', CC' and DD', passing through the collection will show the particles not to be in contact. For real granular soils a similar situation can be expected. This observation can provide the basis for understanding, for example, how rupture propagates through a granular mass.

Conventional soil mechanics failure solutions assume that the rupture surface progresses within the plane of consideration. However, as noted, because there are few contacts to be broken in a given vertical plane, the propagation of rupture, should it continue, is much more likely to occur in the third dimension rather than in the plane of consideration. It would seem

Fig. 1

that similar effects must affect how boundary energy is transmitted through particulate media.

Can the Authors indicate how the distinct element method may be extended to model the more realistic situation existing in real granular assemblies in which only a few particles are in contact in a plane?

Authors' reply

Professor Harr and Professor Holtz express the opinion that the number of particles that are in contact in a plane affects the behaviour of a granular mass, particularly the propagation of rupture surfaces. The Authors would like to emphasize that the technique of numerical testing is intended to verify opinions such as the one expressed by Harr and Holtz, rather than build them into the model. The distribution of contacts in a plane is not used in the method; contacts are formed and broken on the basis of physical laws that govern the interaction of particles.

The model presented in the Paper is a two-dimensional one and suffers from the drawback that all contacts are lines of contacts between rods rather than points of contacts between spheres. The development of a two-dimensional model should be viewed as a first step. An elementary three-dimensional version of the program BALL discussed in the Paper has been developed, and the first results of three-dimensional modelling have been published (see Cundall & Strack, 1979).

Numerical methods of the type used by Dr Burman idealize a granular assembly subjected to a prescribed loading history as a sequence of static, boundary value problems. However, it is not known that a unique set of contact forces and displacements exists for any given boundary value problem of a granular assembly. If such uniqueness does not exist, an implicit, static method will produce solutions that depend on the particular numerical algorithm used, because

there is no physical way in which the algorithm can select the correct path from the several alternatives. Therefore the Authors prefer to use an explicit, time-marching method whereby the move from one state to the next is determined by physical laws of motion. The problem of non-uniqueness may then be studied by varying the physical parameters.

Dr Burman mentions that he prefers to consider regular assemblies because of the conceptual advantage and because continuous stress and strain fields may be defined readily. He further reports that his access to such fields enabled him to demonstrate the independence of stress and strain rate vectors as predicted analytically by de Josselin de Jong (1971). It may be noted that Drescher & de Josselin de Jong (1972) achieved the same result on an irregular assembly of photoelastic discs. They used average stress and strain rate tensors. The Authors utilize similar average tensors for the irregular assemblies modelled by the program BALL (see Strack & Cundall, 1978).

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A computer model for the analysis of ground movements in London clay

SIMPSON, B., O'RIORDAN, N. J. & CROFT, D. D. (1979). *Géotechnique* **29**, No. 2, 149-176.

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It seems that the model proposed by the Authors is able to predict satisfactorily the behaviour of excavations in London clay as well as the initial stages of loading of conventional laboratory tests and of in situ large diameter plate load tests. However, except for the laboratory triaxial and plane strain tests, the accuracy of the predictions of the model relies heavily on the assumption of a threshold of stress (or strain) changes below which the material is assumed to behave elastically with a stiffness equal to ten times that generally quoted for laboratory tests for London clay. The importance of this initial, very stiff, threshold stage is clearly shown in Figs 16 and 17 of the Paper in connection with the parametric study performed by the Authors for the New Palace Yard excavation.

Laboratory triaxial tests have been recently performed in the soil mechanics laboratory of Imperial College specially designed to investigate the behaviour of London clay in the small strain range. Axial displacements have been measured directly on the specimen on its central part, therefore free from end effects and bedding errors. Overall displacements have also been measured directly between the top cap and the pedestal. The equipment is described by Dara-mola (1978) and Costa Filho (1979) but at this stage it is sufficient to note that axial strains of about 0.05% can be measured with an accuracy of $\pm 0.005\%$ in the central part of a specimen 76 mm high.

Block samples were taken from an open pit in South Ockendon, east London. This site has