reason why Shield's equations should not apply to the case treated in the analyses, and note that McCabe does not indicate why he thinks they do not.

The Authors agree that more research is desirable, and the results of further work carried out at Warsaw, Cambridge and Warren Spring are being written up. In addition, P. G. Murfitt at Warren Spring Laboratory proposes to run the experiments McCabe describes in his last paragraph, as well as some of more direct relevance to the transition from mass to core flow in a hopper. Nevertheless, it appears to the Authors that McCabe has not made the case that the assumptions adopted in the Paper are unsatisfactory.

Pressuremeter tests in sands

HUGHES, J. M. O., WROTH, C. P. and WINDLE, D. (1977). *Géotechnique* 27, No. 4, 455–477.

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The Paper is a considerable step forward in the field of in situ determination of sand properties. The following two comments are made for reasons of consistency, not to contest the proposed method.

In the last paragraph on page 462, the Authors state that '... all elements of soil will be subjected to the same stress path ...'. This is an assumption, tacitly made in the Paper, which should be replaced by the assumption ' $\dot{\epsilon}_r/\dot{\epsilon}_\theta = -(1 - \sin \nu)/(1 + \sin \nu)$ holds for all soil elements in one horizontal plane'. As was shown by Gudehus *et al.* (1977), the latter assumption suffices to prove the first one. In addition, this assumption makes sure that the radial stress remains constant once the limiting state of stress is reached (*ibid.*).

This new assumption, referring to $\dot{\epsilon}_r/\dot{\epsilon}_\theta$, is certainly reasonable. It may be supported by radiograph observations and is justified, to a certain extent, by the success of the Authors in their analysis.

Secondly, the evaluation should not be based on simple shear or triaxial, but on biaxial tests. Vardoulakis (unpublished) has shown that ϕ -values cannot be determined from triaxial tests, except for very low stress levels, as the sample bulges before having reached the limiting state. The biaxial test (Vardoulakis, 1977) is ideally suitable for the pressuremeter analysis (plane strain, no rotation of principal axes). The ϕ - and ν -values from such tests are markedly higher, yielding higher K -values. A new evaluation of pressuremeter tests (on this basis) yields consistent ϕ -values.

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The Authors have presented a very interesting contribution to the use and to the analysis of pressuremeter tests in sands. It is of immediate practical application for the self-boring pressuremeters developed in France (Baguelin, Jézéquel, Lemée and Le Méhauté, 1972) and in England (Wroth and Hughes, 1973).

The method presented includes a very concrete example of application of Rowe's (1962)

stress-dilatancy relation and allows for deriving both the angle of shearing resistance ϕ and Hansen's (1958) angle of dilation ν , knowing the angle of internal friction at critical state ϕ'_{ev} . This is achieved by interpreting the pressuremeter test for moderate strains, let us say between 1 to 2% and 10%. Thus, the assumption made by the Authors that volumetric strains increase linearly with shear strains seems reasonable and is supported by experimental data. This point should be noted for comparison with Ladanyi (1963) quoted by the Authors. To determine ϕ' , Ladanyi uses the part of the pressuremeter curve corresponding to shear strains larger than 10%. In this region Ladanyi makes the assumption that dilatancy has reached a constant level ν_t which is also furnished by his trial and error method.

Besides practical methods of determining the properties of sands, it can be of interest to know how dilatancy or volume change affects theoretically the results of pressuremeter tests. Several studies can be mentioned.

For small strains, Frank (1974) proved that an additional volume change due to shear, in comparison with linear isotropic elasticity, does not change the initial tangent of the pressuremeter curve (σ'_r, ϵ) which remains twice that of the elementary shear curve of the soil, i.e. twice the shear modulus G . He showed using the finite element method that the vertical shears created by the insertion of the probe in this linear dilatant soil do not perturb the natural horizontal pressure. This theoretical result is necessary to justify the use of the self-boring pressuremeter in sands, in order to get K_0 and meaningful expansion curves.

Ruppeneit, Bronshtein and Dolgikh (1973) have studied the problem in an elastic soil, anisotropic in compression and traction, which thus also exhibits volume changes due to shears. Though the practical use of their model (Ruppeneit, 1977) still requires questionable hypotheses, interesting theoretical results are given.

Wroth and Windle (1975) have taken into account volume changes related to the circumferential strain ϵ_θ , to derive the elementary shear curve of the soil from the pressuremeter curve.

Ladanyi (1961) studied the influence of volume changes, either by compression or by distortion, on the limit pressure of the expansion of a cylindrical cavity in granular soils. Though his theoretical derivations are not all quite correct for large strains (see Baguelin, Jézéquel and Shields, 1978) it should be emphasized here that he was the first one to point out and to show the importance of deformability properties of sands in the pressuremeter tests results.

The comprehensive theoretical study in Baguelin, Jézéquel and Shields (1978), apart from giving for the first time the correct fields for large strains, includes several interesting findings. Thus, the influence of compressibility or dilatancy for granular soils is clearly shown on the yield and limit pressures. This study also shows how, in any soil and for all magnitudes of strains, the mobilized secant modulus of the shear curve can be derived simply from the pressuremeter curve. This result is necessary to get shear moduli G from the pressuremeter test results. The Authors refer to the linear elastic model in their Paper, when deriving moduli G for the wash sand or the Kernick fill, which is insufficient.

It has been pointed out at the beginning of this contribution that the Authors applied their analysis to small strains data. It is true indeed that two of their relationships, equations (9) and (10), would not hold true at large strains. For instance, in equation (9), the relation between the volume change v and the variables ϵ_r and ϵ_θ would be incorrect. Equation (10) would lead to infinite volume change v at the limit pressure value. When dealing with large strains, the second order terms $d\xi/dr^2$ and $(\xi/r)^2$ cannot be neglected in the basic definitions of strain (equation 4) in order to form strain tensors. Green strains or Almansi strains must be used respectively with Lagrange or Euler coordinates (see Baguelin and Frank, 1976; Baguelin, Jézéquel and Shields, 1978). Again, these remarks do not invalidate the application made by the Authors, since only moderate strains are involved.

It should be mentioned that Ménard (1957) and Cassan (1960), respectively for small strains and for large strains, gave the first practical method for obtaining Mohr's circles and failure envelopes of granular soils from pressuremeter curves, assuming, like Gibson and Anderson (1961), that the soil deformed and failed at constant volume.

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Authors' reply

The Authors are grateful for the favourable and useful comments made both by Professor Gudehus and Messieurs Baguelin and Frank. Regarding those of Gudehus, the Authors agree that ideally the evaluation of the pressuremeter test should be based on the results of plane strain tests in which the principal axes of stress do not rotate, i.e. the biaxial test, and not on the simple shear test or the triaxial test. It is well established that the values of ϕ' and ν are higher in plane strain tests than in triaxial tests, and these would lead to a higher value of ϕ' being interpreted from pressuremeter tests in sand. However, in most practical instances it will only be possible to conduct triaxial tests in order to obtain a value of ϕ'_{cv} for the derivation of ϕ' and ν but this will lead to conservative values of ϕ' for the sand for use in design.

The Authors consider the comments of Baguelin and Frank particularly valuable. They have elaborated on the problem of taking proper account of the dilatancy of sand, and of the

restriction of the proposed interpretation to pressuremeter tests in which the sand is subjected to moderate values of strain. They have provided additional references for the reader, including their own comprehensive book (Baguelin, Jézéquel and Shields, 1978) which has recently been published, and which was not available when the Paper was prepared.

Estimating foundation settlements in sand from plate bearing tests

PARRY, R. H. G. (1978). *Géotechnique* **28**, No. 1, 107–118.

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With the publication of the paper by Sutherland (1963) frequent reference has been made by UK researchers to the 1957 investigation by Gibbs and Holtz (1957) into the determination of the density of sands by means of the SPT.

It has been brought to the attention of the writer by several practitioners that the Gibbs and Holtz corrections to the recorded SPT results to allow for the effects of overburden pressure provide over-estimates of the relative densities of sand deposits. The lack of agreement between research and practice may be attributed to the laboratory conditions not being representative of field conditions (Thorburn, 1963).

Peck has concurred with this latter view and has stated that all relative densities estimated from N values by the Gibbs and Holtz procedure are likely to be greater than the actual relative densities in the field. It would be of great assistance to researchers and practitioners if this twenty year old argument was resolved by proper research.

The conclusions of the British Geotechnical Society Sub-Committee on Penetration Testing (1974) is worthy of reference and the writer would be pleased to receive comments on this important debate from Mr Parry, particularly since he employs the Gibbs and Holtz correlations in his Paper.

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Forssell (1977) made the assumption that one part of the settlement under a loaded plate on sand, a_{el} , was a function of the elasticity of the sand material. Another part, a_ϕ was a function of relative movements between the sand particles, caused by friction.

Thus

$$a_{el} = \sigma_m \frac{\sqrt{A}}{K_{el}}$$

where σ_m = the mean bearing stress

A = the bearing area

K_{el} = a modulus depending on the shape of the bearing plate and on the elasticity of the sand material