$$w = \frac{qb}{2m} \int_0^\infty J_0(r\xi) J_1(b\xi) e^{-\xi z} d\xi = \frac{qb}{2m} I_{202} \quad (5)$$

which reduces on the z axis to

$$w = \frac{q}{2m} [1 - z/\sqrt{(b^2 + z^2)}] \quad (6)$$

The behaviour of non-homogeneous media is of interest not only in relation to ground for which the modulus increases naturally with depth, but also for rock adjacent to excavations for which the degree of induced loosening decreases with distance behind the surface.

Author's reply

This is a result the validity of which I have suspected for a long time and I am very glad that Alexander has finally resolved the matter and that he did not—like me—balk at the algebra. The result is, in some ways, rather surprising for it pays no regard to the well-known doctrine that 'more rigid material attracts stress'. Furthermore, it is of such generality that a search at a deeper level for the nature of the equivalence between the two media should be rewarding: I do not believe it to be wholly fortuitous that the displacement field in 'my' medium is, to within a constant, the z -derivative of the displacement field in the homogeneous elastic half-space.

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Risk analysis of slopes and its application to slopes in Canadian sensitive clays

ALONSO, E. E. (1976). *Géotechnique* **26**, No. 3, 453–472.

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We wish to comment on the Green Creek landslide, the example used by the Author as illustration in his paper. This landslide occurred more than 1100 years ago. Evidence obtained from the examination of a buried soil profile under the apron of the landslide by Eden (1967) indicates that ground conditions were much wetter at the time of the landslide. As most landslides occur in wet seasons, the risk of failure at this site should be estimated using the distribution of the highest annual values of pore-pressure, rather than the mean observed value.

Fig. 1. Strength data for several sites in the Ottawa Area

The strength parameters used in the example were derived from isotropically consolidated undrained triaxial tests. Because of the limitations of this type of test, it has been found that the stress paths meet the failure envelope at a narrow stress range, usually higher than that prevailing in the slope.

In slope stability investigations subsequent to the Green Creek study, it has been found that drained triaxial tests conducted at constant mean normal stress permit failure to be achieved in stress ranges appropriate to the stresses existing in the slope. Fig. 1, reproduced from Eden and Mitchell (1973), presents results from several sites from the Ottawa area. This figure shows that C' is much smaller and ϕ' higher than the values used by the Author in his analysis of the Green Creek landslide. Lefebvre and LaRochelle (1974) using 'residual strength parameters' obtained from CIU tests show C'_r to be 0.07 kg/cm² and ϕ'_r to be 31°, which are in general agreement with those shown in Fig. 1.

The Author has indicated that, of the eight parameters used in the analysis, pore-pressure and cohesion exert a predominant influence in the total variance. If smaller values of C' and higher ϕ' were used, the critical slip circle would have been much shallower than is shown by the Author. Thus the relevant pore-pressures used in the calculations should be derived from the maximum observed values from piezometers at depths of 3 m and 6 m. As these piezometers were open standpipe type, which have a considerable time lag, it is possible that the maximum pore-pressures are higher than the maximum values observed.

Recent studies of slopes in the St Lawrence Valley clays have emphasized the importance to slope stability of geological features, such as presence of buried valleys, artesian pressures, leaching of salt, sloping strata, fissures and other defects in the clay mass. Because of such geological features, some areas are much more prone to landslides than others. We suggest that until such information on a given area is available, a general risk analysis has little real value.

Author's reply

Eden and Law concentrate their discussion on the pore-pressure and strength parameters used in connection with the example which illustrates the application of the method of analysis developed in the Paper. They recommend that the distribution of the highest annual values

of pore-pressure rather than the distribution of the mean observed value should be used to estimate the risk of failure. Actually I did not use any distribution of mean pore-pressure (which should be understood in a statistical sense of parameter estimation). Rather, I used an estimation of the variance of the pore-pressures measured during at least a full year of observations as reported by Jarret and Eden (1970). Mean pore-pressures were assumed for a specific reference configuration of the slope (Fig. 6 of the Paper). The pore-pressure contribution to the total variance of resisting moment is not, however, affected by the assumed mean values but by the variance of pore-pressure as the expression for W_u in the Appendix points out. Even if the piezometer readings consistently underestimate the actual pore-pressures (open standpipe type), the computed variance would be hardly affected provided the bias error has approximately the same absolute value.

The selection of the correct pore-pressure distribution should be focused from another perspective. Fig. 2, taken from a recent paper (Alonso, 1976), is a qualitative view of the safety evolution in time of the slope. The influence of geometry change (external geodynamic effects), decay of soil strength physico-chemical weathering, slow creep and even man-induced actions such as traffic vibration, blasting and so on) and groundwater oscillations is schematically indicated. The combined effect of all these actions will result in a non-uniformly varying mean safety factor F , as Fig. 2(d) indicates. Groundwater oscillations will have a random pattern with seasonal variations but, probably, with a constant mean over several years. No matter what time is chosen to evaluate the risk there will always exist some probability of failure. If the risk of failure during the wet season is investigated, the distribution of pore-pressures observed in the course of time, during this interval, should be introduced into the analysis. There might also be reasons to be interested in knowing the safety margin in other periods. A satisfactory analysis requires therefore the existence of continuous records of pore-pressure fluctuations over several years.

The pore-pressure data used in the Paper represent then (and this is only heuristically justified) an average situation over a year with no specific reference to any one time or season. In any case the available information is far from allowing a correct characterization of extreme values, very sensitive to the behaviour of the distribution tail.

Figure 2 shows that an adequate analysis of the evolution of the risk of failure of slopes requires the assistance of the theory of random processes since the safety factor, F , is now a random function of time. Within this context the risk of failure is typically a level crossing problem.

I share the opinion of Eden and Law that much more data are necessary to establish a general risk analysis (in a sense of wide applicability to spatial and particularly to temporal situations). A good reason for this is that the probabilistic analysis performed here has not any possibility of improving deterministic, mechanistic models but only the ability to provide different and, in a sense, more complete answers. This brings out the question of the pertinent strength parameters used in the example. They were intended to represent, when possible, reported conditions in the previous analysis by Crawford and Eden (1967) in order to compare the deterministic and probabilistic solutions. This comparison followed, basically, two lines of thought.

First, the sensitivity of the risk of failure was investigated with regard to the different uncertainties involved in the problem. These come primarily—and in the case of soil strength—from soil heterogeneity and testing errors which are not affected by the type of strength parameter used. On the other hand, the relative amount of dispersion of parameters is the major reason for the reported sensitivity results. Variations in mean soil conditions have a limited influence in this respect.

Fig. 2. Time effect on slope stability: (a) geometry change; (b) decay of soil strength; (c) groundwater oscillations; (d) combined effect

Secondly, the relationship between (mean) safety factor and risk of failure was investigated. This was actually done by changing arbitrarily the mean soil strength parameters so as to get different safety factors. Therefore, if a deterministic analysis of the slope safety is performed with different mean strength parameters from those chosen in the example (because they are believed to represent better the real behaviour) a different (mean) safety factor would be obtained and, accordingly, a different risk of failure—which would surely be quite similar to the risk indicated in Fig. 16 of the Paper for the corresponding mean safety factor—will be computed. In summary, a conventional analysis of the Green Creek slide performed today, based on strength parameters derived from constant mean stress CIU tests, would have shown a (mean) safety factor different from the one reported by using a more conventional approach but still an acceptable risk of failure consistent with the computed mean safety factor could be derived from Fig. 16.

Many questions remain, however, unsolved in connection with the strength parameters. Eden and Law mention the presence of fissures and other defects in the clay mass. Also relevant are strength anisotropy and rate effects. If their influence is first identified and then quantified a different (mean) analysis will be conducted. If this is not possible, the designer has the last resort of assigning (based only on personal judgement) a global uncertainty to the deterministic stability model or to the strength (or other) parameters in the sense mentioned in the Paper for the case of strength anisotropy.

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Combined active and passive rotational failure of a retaining wall in sand

MILLIGAN, G. W. E. & BRANSBY, P. L. (1976). *Géotechnique* **26**, No. 3, 473–494.

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With the paper of Milligan and Bransby the Cambridge group has made another valuable contribution to increase the knowledge of the stress–strain behaviour of sand. However, it seems that the theoretical interpretation of the test results is influenced too much by the traditional application of the method of characteristics. Moreover, the assumptions, measurements and interpretations concerning the wall friction δ leaves the reader rather confused.

In the present contribution a few points will be made about the assumptions and the attention is drawn to the earth pressure theory of Brinch Hansen (1953) which has been shown to be surprisingly accurate in the prediction of earth pressures.

WALL FRICTION

Apparently the Authors aimed at a smooth wall as the Duralumin was rubbed with successively finer grades of emery paper. It is very doubtful, however, that a Duralumin surface in contact with sand could ever be considered even relatively smooth, as the sand acts as a very efficient coarse emery paper. The recorded wall friction distribution measured by the load cells, which presumably have the same surface as the rest of the wall, bear out that the wall is at any rate relatively rough. The measurements surely leave the Authors in a dilemma as they state that the mobilized value of δ measured by the authors was about 15° even though the wall itself was much smoother. Either the value of δ corresponds to the roughness or the load cell readings are misleading. In this context it is very important to realize that the mobilized angle of wall friction can obtain any value $-\varphi \leq \delta \leq \varphi$ depending only on the state of strain. Even if the sand in contact with the wall behaves like a rigid body the equals signs are allowed (the body is on the point of sliding). Reasonably smooth surfaces might be obtained if the wall and load cells were fitted with a glass cover combined with layers of silicone grease and rubber membranes as is used in triaxial tests with smooth pressure heads.

MODE OF FAILURE

Two different failure mechanisms were observed: a so-called ‘deep failure’ for positions of the axis of rotation near the foot of the wall, and ‘shallow failure’ for higher positions. In my opinion this is merely a question of changes in the original geometry of the tests—an important