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The consolidation of soils exhibiting creep under constant effective stress

GARLANGER, J. E. (1972). *Géotechnique* **22**, No. 1, 71–78.

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Based on the soil model described by Bjerrum (1967), Garlanger has expressed the void ratio change corresponding to an increase of effective pressures from p_0 to σ' , according to the strain path OAB when $\sigma' \leq p_c$, and OCDE when $\sigma' \geq p_c$ (Fig. 1). In the latter case, at a time t after the end (t_1) of the increase of effective stress up to p_t , the void ratio change has been presented by the equation

$$\frac{e}{e_0} = \left(\frac{p_c}{p_0}\right)^{-a} \left(\frac{p_t}{p_c}\right)^{-b} \left(\frac{t_1+t}{t_1}\right)^{-c} \quad (1)$$

a and b being the respective slopes of the lines OC and CD. Garlanger considers that the stress $p_t = p$ is a time dependent variable. Consequently

$$\frac{\partial e}{\partial t} = \frac{\partial e}{\partial p} \frac{\partial p}{\partial t} + \left(\frac{\partial e}{\partial t}\right)_c \quad (2)$$

where

$$\frac{\partial e}{\partial p} = \frac{be}{p} \quad (3)$$

and

$$-\left(\frac{\partial e}{\partial t}\right)_c = \left(\frac{ce}{t_1}\right) \left(\frac{e}{e_0}\right)^{1/c} \left(\frac{p}{p_0}\right)^{b/c} \quad (4)$$

with $p \equiv \sigma'$ according to the notation in Fig. 1. When $p \leq p_c$, equation (1) reduces to

$$\frac{e}{e_0} = \left(\frac{p}{p_0}\right)^{-a} \left(\frac{t_1+t}{t_1}\right)^{-c} \quad (5)$$

In this case, Garlanger has obtained

$$-\frac{\partial e}{\partial p} = \frac{ae}{p} \quad (6)$$

However, at $\sigma' \leq p_c$ the time t_1 has to be considered a stress dependent parameter and so equation (3) is regained for $\partial e/\partial p$.

If one assumes a sudden effective stress increase, the slope a becomes zero and the strain paths OAB and OCDE have to be replaced by respective strain paths OFB and OGE (Fig. 1).

Fig. 1. Time lines

Extrapolating the time lines

$$e = [e(\sigma')]_{t_1 = \text{constant}}$$

beyond the 'instant line', the void ratio change, in both cases, can be expressed only by the equation

$$\frac{e}{e_0} = \left(\frac{t'}{t'_1}\right)^{-c} \quad (7)$$

where

$$t' = t'_1 + t \quad (8)$$

and, according to Fig. 1,

$$t'_1 = t'_0 \left(\frac{p_0}{\sigma'}\right)^{1/d} \quad (9)$$

Thus equation (7) can be written

$$e = (At'\sigma'^{1/d})^{-c} \quad (10)$$

A being an invariant of the (e, σ', t') system

$$A = (e_0^{1/c} p_0^{1/d} t'_0)^{-1} = (e^{1/c} \sigma'^{1/d} t')^{-1} \quad (11)$$

If the stress σ' is considered a time dependent variable, the differentiation of equation (10) yields

$$-\dot{e} = ce \left(A e^{1/c} \sigma'^{1/d} + \frac{1}{d} \frac{1}{\sigma'} \frac{\partial \sigma'}{\partial t'} \right) \quad (12)$$

Considering that

$$\frac{b}{c} = \frac{1}{d} \quad (13)$$

and

$$(t_1 e_0^{1/c} p_0^{b/c})^{-1} = A \quad (14)$$

the first member of equation (12) is identical to Garlanger's equation (4), and the second member identical to $(\partial e / \partial p)(\partial p / \partial t)$ with $\partial e / \partial p$ according to equation (3).

Now, the validity of the respective equations (1) and (5), and (7) and (10) was experimentally verified only under the condition

$$\frac{\partial p}{\partial t} = \frac{\partial \sigma'}{\partial t'} \rightarrow 0 \quad (15)$$

Fig. 2. Isotaches

In this case, the second member in equation (12) disappears and the remaining term

$$-\dot{e} = cAe^{(c+1)/c}\sigma'^{1/d} \quad (16)$$

represents the 'isotache' set (Šuklje, 1957) of the special rheological scheme according to Fig. 1. This set is shown in Fig. 2. By introducing equation (16) into the Terzaghi-Biot differential equation of consolidation, the solution is obtained for arbitrary boundary conditions concerning: (a) the initial (e, σ') values varying with the coordinate z ; (b) the magnitude of the applied load and (c) the speed of the load application. Examples of such numerical analysis of the consolidation process have been given in several publications (Šuklje and Kogovšek, 1968; Šuklje, 1969a and b; Battelino, 1970; Šuklje and Simončič, 1972; Šuklje and Kozak, 1972) and different expressions $e = e(\dot{e}, \sigma')$ for isotaches have been used corresponding to deformability tests of various specimens.

For the construction of isotaches the time lines $e = [e(\sigma')]_{t_1 = \text{constant}}$ (Fig. 1) are not needed. They can be obtained from the consolidation lines (preferably from secondary consolidation branches) in the way explained in previous publications (Šuklje, 1957 and 1969a). To avoid structural disturbance a slow, continuous load application is recommended interrupted by the long-term observation of the secondary consolidation at certain total stresses σ .

Berre and Iversen (1972; companion paper) have found good agreement of experimental time-compression curves with theoretical curves according to the theory of Barden (1965) in which the soil skeleton is represented by a spring coupled in parallel with a non-linear dashpot. The predicted pore pressures were, especially for the lowest specimens, too low at small values of time; they are better, the higher the specimen. The rheological relationships of the non-linear Kelvin model of Barden's theory can be expressed by parallel straight isotaches

$$e = [e(\sigma')]_{\dot{e} = \text{constant}} = (e_2 + a\sigma) - a\sigma' - be^{0.1/n}$$

and

$$e_2 + a\sigma = \text{constant}$$

The speed of the effective stress change $\dot{\sigma}'$ does not influence the consolidation process. According to Berre and Iversen's comparison of experimental and theoretical consolidation curves, the speed $\dot{\sigma}'$ does not seem to affect the consolidation of natural layers of soil whose thickness is of the order of magnitude of 1 m or more and whose properties are similar to those treated in both companion papers.

Garlanger requires that right after a sudden load application the additional effective stresses should be zero ($\mu_1 = 1.0$), except at the drainage boundary. In the case of viscous soils, this requirement is not justified. It seems to be because of a wrong application of the classical

principle of effective stresses to viscous soils. Whether, after a sudden load application, the consolidation will start at zero additional effective stresses depends on the initial porosity (consolidation speed respectively), the permeability, the loading interval and the drainage path (layer thickness). The interpretation, proofs and examples of the above statement have been given elsewhere, e.g. Šuklje, 1957, 1969 (a and b); Šuklje and Simončič, 1972.

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J. Garlanger

The Writer raises two questions concerning the Paper which are as follows.

- (a) What is the correct compressibility to use in determining the change in pore pressure with time during the consolidation process?
- (b) How is the excess pore pressure that is induced in a saturated soil by one-dimensional, instantaneous loading related to the applied total stress?

The answer to the first question is of paramount importance because a correct value for the compressibility of the soil structure is essential to the analysis. The solution to the following equation

$$\frac{\partial e}{\partial t} = \frac{k(1+e_0)}{\gamma_w} \frac{\partial^2 \bar{u}}{\partial z^2} \quad (1)$$

(which gives the rate of consolidation at any point within a soil mass free to drain in only one direction) is not possible without obtaining the distribution of pore pressures throughout the soil mass as a function of time during the entire consolidation process.

As discussed in the Paper, the change in pore pressure with time can be given as

$$-\frac{\partial \bar{u}}{\partial t} = \frac{\frac{\partial e}{\partial t} - \left(\frac{\partial e}{\partial t}\right)_c}{\frac{\partial e}{\partial \bar{p}}} \quad (2)$$

where $\partial e / \partial t$ is the consolidation rate given by equation (1), $(\partial e / \partial t)_c$ is the rate of creep under constant effective stress and $\partial e / \partial \bar{p}$ is the compressibility of the soil structure. It can be seen from equation (2) that the rate of dissipation of pore pressure during consolidation is controlled

by the creep rate and the compressibility of the soil structure. These two parameters must be determined together, i.e. they are not independent.

The Writer suggests that the creep rate and the consolidation rate may be set equal to each other giving $\partial u / \partial t \rightarrow 0$. This is certainly possible and probable during part of the consolidation process but is not generally true and should not be the basis for a solution of equation (1).

Consistent values of compressibility and creep rate are given by the Author's equations (8), (9) and (10) (the Writer's equations (6), (3) and (4) respectively). Equations (8) and (9) were obtained by assuming that a change in void ratio caused by a change in effective stress would occur as soon as the effective stress changed, i.e. at $t=0$.

However, it was realized after several solutions had been obtained, that equation (8)

$$\frac{\partial e}{\partial p} = \frac{ae}{\bar{p}}$$

predominantly controlled the consolidation process, i.e. only the soil elements at the drainage boundary were ever loaded beyond the critical pressure, where the compressibility given by equation (7) was applicable. This is not surprising, because as the soil consolidates, it creeps and develops a reserve resistance to further compression (Bjerrum, 1967).

As it is not possible to separate creep from compressibility during a consolidation test, it is not possible to obtain exact values for the slope a of the instant line between p_0 and p_c . The values used in the calculations were, of necessity, approximate. (See section on example of calculation for a discussion of how a was chosen.) However, it is certainly not possible to assume, as the Writer suggests, that the soil structure is infinitely rigid ($a=0$). A compressibility of zero would give a dissipation rate approaching infinity.

The concept of soil compressibility is important in the second question raised by the Writer concerning the relationship between excess pore pressure, Δu , and applied total stress, $\Delta \sigma$, for the conditions of one-dimensional, instant loading. This relationship, commonly expressed as the ratio $\Delta u / \Delta \sigma$, is given by the following equation (see Skempton, 1954 for its derivation)

$$\mu = B = \frac{1}{1+n(c_w/c_s)} \quad \dots \dots \dots (3)$$

where n is the porosity of the soil skeleton, c_w is the compressibility of water, and c_s is the compressibility of the soil structure. It is never possible for B to have a value exactly equal to 1.0. The use of this value for μ in the calculations was only done for convenience. The theoretically correct value, with $a=0.010$, $p_0=1 \text{ kg/cm}^2$ and $e_0=1.4$, would have been 0.9977.

It is also recognized that it is impossible to measure a value of B exactly equal to the theoretically correct value. Such factors as the compressibility of the measuring system, leakage at the connexions, and air in the measuring system can all cause significant time dependent differences between measured and calculated pore pressures. (For a discussion of pore pressure measurements in soils, see Wissa (1968).)

However, it should be mentioned that the assumptions of instantaneous loading, uniform distribution of pore pressure throughout the specimen and a B coefficient essentially equal to 1.0 were met by the laboratory specimens used to check the Author's calculations. However, it would have been possible and not difficult to include time dependent loading and non-uniform pore pressure distribution in the calculations if these conditions had been important.

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