DISCUSSION

Theoretical description of embankment erosion owing to overflow

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I have read this paper with considerable interest because of our involvement in studying the effects of: (a) flood water flowing over earthfill dams (overtopping); and (b) flood water releases through unlined earthen spillways. Specifically, our interests are in the erosional and sliding stability of earthfill and rockfill dams owing to overtopping flows during extreme flood events; and in erosional and headcutting stability of unlined earthen spillways during large releases of flood water events when depth and duration of overflows may be significant. Erosion, sliding stability and headcutting (individually and collectively) are affected by the tractive (hydrodynamic) forces on the channel boundaries (bed and side slopes) exerted by the overflow. This discussion is primarily for the determination of tractive forces on the erosional surface (crest and downstream face) of an embankment dam; a few other comments are included.

Embankment dams generally have flatter upstream slopes as compared with downstream slopes – the rationale being high pore water pressure causing slope stability concerns during rapid drawdown of the reservoir. However, the embankment model in the experiment has the opposite slope geometry; that is, the upstream face has a 1H:1V slope and the downstream face has a 4H:3V slope. Hydraulic characteristics of overflow (pressure, velocity, traction) are likely to be affected by the upstream slope of the embankment. It would be helpful to know: (a) the reasons for adopting the model geometry which is different from the one commonly used in practice; and (b) if the results presented in the paper are considered applicable to real embankment dams and unlined earthen spillways.

For an embankment overtopping, the critical locations requiring protection against erosion and headcutting are the upstream and downstream edges of the dam crest (smooth transition from upstream to downstream slope), the abutment groin areas and the downstream toe of the dam. For an unlined earthen spillway with ogee control structure, the critical locations requiring protection against erosion and headcutting are the spillway chute (the floor and sidewalls including the floor–wall junctions) and the toe where the hydraulic jump forms and energy dissipation occurs. It would be helpful to know if the authors' observations of model tests support these critical locations or suggest other areas of potential concern. It is understood that in addition to the geology, configuration and roughness of the flow surface are the primary contributors affecting the outcome owing to hydraulic overflow.

In equations (1) and (2) of the paper, parameter L is identified as the representative flow length: L_m in the model and L_p in the prototype. However, in comparing the test results with those of Hanson *et al.* (2005) – treating Hanson *et al.* results as prototype – L is used as the dam height. It would be helpful to know: (a) the precise definition for the parameter L in equations (1) and (2); and (b) why the dam height and not the downstream slope length was selected in making the comparison of test results with those of Hanson *et al.*'s (2005) models. Also, it would be useful to know

what kind of validations of experimental findings with actual spillway or dam failures (owing to overtopping flows), if any, were performed and what the results were.

In open channel hydraulics, the tractive force for non-uniform flow is commonly computed using the formula

$$T = \gamma D \left[\left(\frac{q^2}{g D^3} - 1 \right) \frac{dD}{dx} + S \right] \quad (61)$$

where γ is the unit weight of water, D is the depth of flow, q is the unit discharge, g is the acceleration owing to gravity and S is the channel slope. Analytical derivation of equation (61) is included in Chugh (1993) and Smerdon (1959). There have been difficulties concerning the sign of the slope term, S . In hydraulic engineering, the slope of a channel is considered positive when the channel slopes downwards in the direction of flow. However, this sign convention is not in accordance with the coordinate axis system used in developing equation (61). Thus, in using equation (61), the slope is negative for channels which slope downwards in the direction of flow. It will be helpful to know if the authors considered the slope to be positive or negative.

Finally, the reference to Temple, G. J. & Moore, J. S. (1997) should read Temple, D. M. & Moore, J. S. (1997).

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- Hanson, G. J., Cook, K. R. & Hunt, S. L. (2005). Physical modeling of overtopping erosion and breach formation of cohesive embankments. *Trans. ASABE* **48**, No. 5, 1783–1794.
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Authors' reply

The authors thank the discussor for his constructive comments; the replies to his comments are written below.

In our model experiment, we had to make the embankment model with the steep upstream and downstream slopes because of the limitation of the size of the experimental apparatus. The reason why the upstream slope was steeper than the downstream one is as follows: when the embankment height decreases owing to erosion, the impounded water, added to the supplied water, discharges. We intended to minimise the discharge of the impounded water, which was difficult to measure.

This experiment focused mainly on the erosion process of the embankment and the slope stability could not be considered because the embankment size was even smaller than that in practice. Therefore, the application of the test results should be limited only to the erosion on embankments.

Although our paper did not deal with the protection against embankment erosion, the protection on the crests of embankments is thought to be quite effective because it can prevent the embankment height from decreasing owing to

erosion, avoid the additional discharge of the impounded water and delay the failure process.

In order to consider the similarity rule on embankment erosion, the following two equations must be considered

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial h}{\partial x} = -g \frac{\partial \eta}{\partial x} - \frac{C_f u^2}{h} \quad (62)$$

$$\frac{\partial \eta}{\partial t} = -\frac{E}{1 - \lambda_p} \quad (63)$$

where u , h , η , x , t are the depth-averaged flow velocity, the flow depth, the height of erosion surface, the horizontal coordinate and time, and E , g , λ_p denote the erosion rate of soils, the gravitational acceleration, the porosity of soils subjected to erosion. Equations (62) and (63) describe the one-dimensional momentum conservation of open channel flow and the temporal change of the height of the erosion surface respectively. They correspond to equations (19) and (24) in our paper if the flow is quasi-steady and the angle of the erosion surface is sufficiently small.

Let the representative length, velocity and time be L , U and T respectively. The representative length L is usually taken as the length which is directly related to the size of the phenomenon of our interest. Using these representative values, equation (62) is non-dimensionalised as follows

$$\frac{U}{T} \frac{\partial u'}{\partial t'} + \frac{U^2}{L} u' \frac{\partial u'}{\partial x'} + g \frac{\partial h'}{\partial x'} = -g \frac{\partial \eta'}{\partial x'} - \frac{U^2 C_f u'^2}{L h'} \quad (64)$$

where u' , h' , η' , x' and t' are non-dimensional variables of u , h , η , x and t . Applying equation (64) to a prototype, the following equation is obtained (note that C_f is non-dimensional and is not dependent on scale sizes in this analysis)

$$\frac{U_p}{T_p} \frac{\partial u'}{\partial t'} + \frac{U_p^2}{L_p} u' \frac{\partial u'}{\partial x'} + g_p \frac{\partial h'}{\partial x'} = -g_p \frac{\partial \eta'}{\partial x'} - \frac{U_p^2 C_f u'^2}{L_p h'} \quad (65)$$

where the subscript p implies prototype. Similarly, the following equation is obtained for a model

$$\frac{U_m}{T_m} \frac{\partial u'}{\partial t'} + \frac{U_m^2}{L_m} u' \frac{\partial u'}{\partial x'} + g_m \frac{\partial h'}{\partial x'} = -g_m \frac{\partial \eta'}{\partial x'} - \frac{U_m^2 C_f u'^2}{L_m h'} \quad (66)$$

where the subscript m implies model. The model and the prototype are dynamically similar only when equations (65) and (66) are equivalent. To satisfy this condition, the following equations are derived, comparing each term of equations (65) and (66)

$$\frac{U_r}{T_r} = \frac{U_r^2}{L_r} = g_r = g_r = \frac{U_r^2}{L_r} \quad (67)$$

where

$$L_r = \frac{L_p}{L_m}, \quad U_r = \frac{U_p}{U_m}, \quad T_r = \frac{T_p}{T_m}, \quad g_r = \frac{g_p}{g_m} \quad (68)$$

From the second equality in equation (67), the Froude-similarity rule

$$\frac{U_r}{\sqrt{g_r L_r}} = 1 \quad (69)$$

is obtained. With the aid of equation (69), the first equality in equation (67) is reduced to

$$T_r = \sqrt{\frac{L_r}{g_r}} \quad (70)$$

which is the temporal similarity ratio under Froude similitude.

In addition to equation (62), equation (63) needs to be considered when erosion is of interest. Assuming that the erosion rate has the following form (see equation (23) in our paper)

$$E = \alpha \left(\frac{u^2}{u_t^2} - 1 \right)^\gamma \quad (71)$$

equation (63) is non-dimensionalised into the following with the representative values

$$\frac{L}{T} \frac{\partial \eta'}{\partial t'} = -\frac{\alpha}{1 - \lambda_p} \left(\frac{u'^2}{u_t'^2} - 1 \right)^\gamma \quad (72)$$

Taking the similar procedure to derive equation (67), the following relationship between the prototype and the model is yielded (note that λ_p is non-dimensional and is not dependent on scale sizes)

$$\frac{L_r}{T_r} = \alpha_r, \quad \alpha_r = \frac{\alpha_p}{\alpha_m} \quad (73)$$

Since equation (73) needs to be consistent with equation (70), α_r must satisfy the following relationship

$$\alpha_r = \sqrt{g_r L_r} \quad (74)$$

If equation (74) is not satisfied, the temporal similarity ratio differs between the open channel flow and the elevation change of the erosion surface owing to erosion. However, the relationship of equation (74) is not necessarily fulfilled between the model and the prototype. When a laboratory model test is compared with a large-scale test, sufficient attention should be paid to the erosion rates of the soils used in the experiments.

In our paper, we could not fully discuss the similarity between our model test and the large-scale experiments by Hanson *et al.* (2005) from the viewpoint of erosion, because we could not comprehend the erosion characteristics of our or their embankment material. We adopted the embankment height as the representative length L because the slope angle of our model embankment was different from that of Hanson *et al.* (2005) and the embankment height better indicated the scale size than the slope length.

In the analyses of our paper, the slope gradient ($= \tan\theta$ or $\sin\theta$) is positive when the channel slopes downward in the direction of flow, as shown in equation (20) or Fig. 7 in the paper.

REFERENCE

- Hanson, G. J., Cook, K. R. & Hunt, S. L. (2005). Physical modeling of overtopping erosion and breach formation of cohesive embankments. *Trans. ASABE* **48**, No. 5, 1783–1794.