Discussion: Consolidation of clayey gouge amid permeating rock masses

Ashutosh Trivedi PhD

Professor and Head, Department of Civil Engineering, Delhi Technological University, Delhi, India

Tanmay Banik BEng

GET, Triune Energy Services Pvt Ltd, New Delhi, India

Tanay Sukumar BEng

Fellow, Teach for India, Green Park Extn, New Delhi, India

Naresh Kumar MTech

Associate Professor, Department of Civil Engineering, Delhi Technological University, Delhi, India

Akshay Jain BEng

Sales Officer, Indian Oil Corporation Ltd, Ranchi, Jharkhand, India

Amit Kumar BEng

Officer Trainee, Indian Oil Corporation Ltd, Vijayawada, Andhra Pradesh, India

S. K. Tewatia

Central Soil and Materials Research Station, Government of India, New Delhi, India

A. Sridharan

Professor Emeritus, Department of Civil Engineering, Indian Institute of Science, Bangalore, India

Subhash Narayan Singh

Engineer, BSNL, Patna, India

K. Tewatia

Department of Civil Engineering, National Institute of Technology, Kurukshetra, India

A. Tewatia

Ansal Institute of Technology, GGS Indra Prasth University, Gurgaon, India

S. Rath

Proctor, University of Engineering and Management, Jaipur, India

Contribution by S. K. Tewatia, A. Sridharan, Subhash Narayan Singh, K. Tewatia, A. Tewatia and S. Rath

The original paper by Trivedi *et al.* (2014) presented a natural complicated shaped in situ clay idealised as a saturated rectangular body settling in a vertical direction with two-dimensional drainage in vertical and horizontal directions. Theoretical equations were derived for two-dimensional drainage using Fourier series. The discussers suggest a more general theoretical equation for three-dimensional drainage when the soil is stratified in X , Y and Z directions.

Theoretical equation for three-dimensional drainage when the soil is stratified in X , Y and Z directions

The basic three-dimensional (3D) consolidation equation in Cartesian coordinates (Biot, 1941) is given as

$$47. \quad \frac{\partial u}{\partial t} = c_x \frac{\partial^2 u}{\partial x^2} + c_y \frac{\partial^2 u}{\partial y^2} + c_z \frac{\partial^2 u}{\partial z^2}$$

where u is pore pressure, t is time, c_x , c_y and c_z ($= c_v$) are coefficients of consolidation in the X , Y and Z directions, respectively.

Suppose, L , B and H are the length, breadth and height of a (cuboid) sample draining outwards from all the six phases and settling in Z or vertical direction only. The Carillo (1942) type solution of Equation 47 is given by Tewatia (2013b) as

$$48. \quad (1 - U_{xyz}) = (1 - U_x)(1 - U_y)(1 - U_z)$$

and the Terzaghi (1923) type solution of Equation 1 is given by Tewatia (2010, 2013a) as

$$49. \quad U = 1 - \frac{8}{\pi^2} \sum_{N=0}^{\infty} \frac{1}{(2N+1)^2} \epsilon^{-\frac{(2N+1)^2 \pi^2 \lambda T}{4}}$$

where λ is given as

$$50. \quad \lambda = 1 + \frac{k_y H^2}{k_z B^2} + \frac{k_x H^2}{k_z L^2}$$

and

$$51. \quad T = \frac{c_v t}{H^2}$$

where k_x , k_y and k_z ($= k_v$) are the permeability of soil in the X , Y and Z directions, respectively. Equation 49 shows that in 3D consolidation, T for a particular t is increased by λ times or c_v is increased by λ times (Tewatia, 2010, 2013a). Hence, U for a particular t increases. Equation 49 can be reduced to a two-dimensional (2D) or one-dimensional (1D) space by putting k_x or k_y or both equal to zero in Equation 50.

From Equation 49, it can be shown (Tewatia, 1998, 2010, 2013a; Tewatia *et al.*, 2007, 2013a, 2014) that a plot of settlement (δ) against rate of settlement ($d\delta/dt$) gives Equation 52 after 60% U .

$$52. \quad \delta = -\frac{4H^2}{\pi^2 \lambda c_v} \left(\frac{d\delta}{dt} \right) + \delta_{100}$$

Equation 52 is the straight line equation in the form $y = m_{3d}x + c$, where m_{3d} is the slope and c is the intercept on the y -axis. The δ -($d\delta/dt$) plot is a straight line having a slope, $m_{3d} = (4H^2)/(\pi^2\lambda c_v)$ and intercept on δ axis, $c = \delta_{100}$. Thus, c_v can be evaluated as

$$53. \quad c_v = c_z = \frac{4H^2}{\pi^2\lambda m_{3d}}$$

$$54. \quad c_y = \frac{k_y}{k_v} c_v$$

$$55. \quad c_x = \frac{k_x}{k_v} c_v$$

The c_x , c_y , c_z and δ_{100} can be evaluated when the settlement, time and pressure at the instant of load increment are not known or the unknown pressure increases non-uniformly with settlement and time in the beginning. This is the usual case in consolidation of clayey gouge amid permeating rock masses. The space between the crack in rocks may be found to be decreasing in the field due to 1D, 2D or 3D consolidation of stratified soil in between the crack without knowing any history. Equations 52–55 can be invoked by plotting the space between the crack against the rate of decrease of crack. For this purpose, a simple method of 3D crack monitoring can be used as suggested by Tewatia *et al.* (2006).

For clay that may contain some sand stream at the centre, the flow may be radial as well as vertical. This is another form of 3D drainage. Equation 49 in this case reduces to

$$56. \quad U = 1 - \frac{8}{\pi^2} \exp\left(-\frac{\pi^2\lambda t}{4}\right)$$

Equation 56 is valid after 60% U (Tewatia, 2010, 2013a; Tewatia *et al.*, 2013b), but the λ is given as

$$57. \quad \lambda = 1 + \frac{32k_r H^2}{\pi^2 k_v d_e^2 F_n}$$

where

$$58. \quad F_n = \frac{n^2}{n^2 - 1} \ln(n) - \frac{3n^2 - 1}{4n^2}$$

where k_r is the radial permeability and n is the drain spacing ratio, given by

$$59. \quad n = d_e/d_w$$

where d_e is the diameter of influence (i.e. twice the effective radial drainage path) and d_w is the diameter of the drain or sand stream.

Equation 56 also gives Equation 52 (with λ by Equation 57) that is a straight line on the δ -($d\delta/dt$) plot. The intercept on the δ axis is δ_{100} , and c_v can be evaluated by Equation 53 and

$$60. \quad c_r = \frac{k_r}{k_v} c_v$$

The shapes of 3D consolidation curves for various values of λ are given by Tewatia (2013a). Here, Equations 52, 53, 57 and 60 can also be used simply by plotting the space between the crack against the rate of decrease of crack without knowing any history of settlement, time and pressure.

New factor λ_{cH}

So far, the shapes of clay draining in 3D to be considered have been very simple, (e.g. cuboid, cylindrical, etc.) having well-defined L , B , H , d_e and r . But suppose the shape is conical or obliquely sliced conical; then the problem becomes more cumbersome. Moreover, the shape of the in situ clay sample can be of any type, whatever one can imagine, for example zodiac signs. To take care of this problem, a new factor λ_{cH} is introduced here, where

$$61. \quad \lambda_{cH} = \frac{\lambda c_v}{H^2}$$

While λ takes care of simple geometric shaped clay, λ_{cH} takes care of complicated shaped clay samples with 3D drainage. The λ_{cH} is thus the property of in situ clay lamina of definite size, shape and drainage conditions, and it can be determined in the field itself (by Equation 52 along with δ_{100}) by observing the trend of settlement for some time without knowing any history, shape, size and drainage conditions. Once λ_{cH} and δ_{100} are known, then other consolidation characteristics, present, past and future time-settlement data and so on can be known using Tewatia's (2010, 2013a) methods.

Secondary consolidation

If the clayey gouge consists of peat or organic soil, then secondary consolidation may occur and Terzaghi's theory cannot be used. When settlement and time are not known in the beginning of load increment, the usual δ -log(t) method of secondary consolidation available in literature fails, as the δ -log(t) graph cannot be plotted because neither δ nor t is known (Tewatia *et al.*, 2007). But as in the case of primary consolidation, $d\delta/dt$ can be plotted against arbitrary settlement (say the space in the direction of settlement between the crack). The magnitude of slope of the δ -log(t) and δ -log($d\delta/dt$) plots is the same. The settlement and time at the beginning of load increment can be evaluated by Tewatia's (2013a) methods.

Determination of in situ stress in rock

Earlier in the case of primary and secondary consolidations, unknown settlement and time at the beginning of consolidation were determined just by looking at the trend of settlement in the field without taking any sample in the laboratory. However, for determination of unknown pressure on clay, which is equal to the stress on walls of the rock pressing the gouge, an undisturbed sample is taken out. Its ultimate settlements are determined with increasing pressures. Then initial pressure is calculated that is equal to the one on the clay in the crack undergoing consolidation. Theory and a solved example of this method are given below.

Theoretical considerations

It is known that for a fairly good range, the $\delta\text{-}\ln(\sigma')$ plot is a straight line, where σ' is effective pressure on clay. Let the straight line portion in $\delta\text{-}\ln(\sigma')$ plot be represented by the following equation:

$$62. \quad \delta = a \ln(\sigma') + b$$

where a is slope of the straight line and b is the intercept on the δ axis. Differentiating with respect to σ' , we get

$$63. \quad \frac{d\delta}{d\sigma'} = \frac{a}{\sigma'} \quad \text{or} \quad \sigma' = a \frac{d\sigma'}{d\delta}$$

Substituting this σ' in Equation 62, we get

$$64. \quad \delta = a \ln\left(\frac{d\sigma'}{d\delta}\right) + (b + a \ln a)$$

The following inferences can be drawn from Equations 62, 63 and 64:

- Comparing Equation 62 with Equation 64: If the $\delta\text{-}\ln(\sigma')$ plot is a straight line, then the $\delta\text{-}\ln(d\sigma'/d\delta)$ plot is also a straight line, having the same slope. The difference between the intercepts of these two on the δ axis is equal to $a \ln(a)$, and on $\ln(d\sigma'/d\delta)$ axis, it is $\ln(a)$. Therefore, these two are parallel lines separated horizontally by a distance $\ln(a)$. These two will also remain parallel on $\log_{10}$ scale.
- Unknown, pressure, σ' , can be evaluated from Equation 63.
- However, a (and hence compression index, C_c) can be determined from the $\delta\text{-}\ln(d\sigma'/d\delta)$ plot without knowing the initial pressure.

Solved example

The suggested method is illustrated by a solved example below. The data have been taken from Table 10, which shows the real pressure-compression curve in the laboratory (Tewatia, 2010).

σ' : kPa	20	50	100	200	400	800
Dial gauge reading	152	260	424	578	730	851

Table 10. Dial gauge readings with known initial pressure

Initial unknown pressure is (say) 20 kPa. So Table 11 has been made after subtracting 20 kPa from all the σ' values.

Question: There was an unknown overburden pressure, σ_0 , on a clayey gouge amid permeating rock. The undisturbed clay specimen was taken in the laboratory and the first dial gauge reading was fixed at 152. Furthermore, pressure increments were given, and final settlements corresponding to them were recorded, which are given in Table 10. Find the unknown overburden pressure and draw the $\delta\text{-}\log(\sigma')$ curve. LC of dial gauge = 0.01 mm.

Answer: In Table 12, $d\sigma'/d\delta$ (Column 6) is the ratio of the difference of two subsequent pressure increments to the difference of two subsequent dial gauge readings. This ratio is taken at the average of two subsequent dial gauge readings (Column 8) and plotted on a semi- $\log_{10}$ graph as shown in Figure 6. A curve is obtained. It is seen from this graph that a perfect straight line appears by joining three middle points. The equation of this line is

$$65. \quad \delta = 213.3 \ln\left(\frac{d\sigma'}{d\delta}\right) + 594.7$$

Here, the slope, $a = 213.3$ on $\ln$ scale or $m = \ln(10)a = 491.3$ on $\log_{10}$ scale. A parallel line at the horizontal distance $a = 213.3$ is drawn. This line corresponds to the linear portion of the $\delta\text{-}\sigma'$ curve on the semi-log scale. The equation for this line is:

$$66. \quad \delta = 213.3 \ln \sigma' - 550.1$$

Equation 66 gives $\sigma' = 197.3$ kPa corresponding to $\delta = 578$. But in Table 11, corresponding to $\delta = 578$, pressure increment = 180. Therefore, the in situ stress $\sigma_0 = 197.3 - 180 = 17.3$ kPa. This correction is applied to all σ' values in Table 12, and final $\delta\text{-}\sigma'$ values are obtained. Now all these corrected values are plotted against σ'

S. No.	σ' : kPa	Dial gauge reading
1	σ_0	152
2	$\sigma_0 + 30$	260
3	$\sigma_0 + 80$	424
4	$\sigma_0 + 180$	578
5	$\sigma_0 + 380$	730
6	$\sigma_0 + 780$	851

Table 11. Dial gauge readings with unknown initial pressure

S. No.	σ' : kPa	DGR	$d\delta = (\delta_n - \delta_{n-1})$	$d\sigma' = (\sigma'_n - \sigma'_{n-1})$	$d\sigma'/d\delta$	$a(d\sigma'/d\delta)$	$\delta = (\delta_n + \delta_{n-1})/2$	Calc. σ' for Col 3: kPa	Actual σ' : kPa
1	σ_0	152						17.3	20
2	$\sigma_0 + 30$	260	108	30	0.2778	59.3	206	47.3	50
3	$\sigma_0 + 80$	424	164	50	0.3049	65.1	342	97.3	100
4	$\sigma_0 + 180$	578	154	100	0.6494	138.5	501	197.3	200
5	$\sigma_0 + 380$	730	152	200	1.3158	280.7	654	397.3	400
6	$\sigma_0 + 780$	851	121	400	3.3003	704.1	790	797.3	800

Table 12. Evaluation of δ – σ' data by suggested method

Figure 6. Evaluation of δ – σ' plot from δ – $d\sigma'/d\delta$ plot

in Figure 6. Actual values are also plotted for comparison. It is seen that calculated and actual δ – σ' curves are almost the same.

Tewatia (1998, 2010, 2013a; Tewatia *et al.*, 2007, 2013b) showed that in primary compression with whatever complicated drainage, shape and size of clay in whatever odd loadings, the δ –($d\delta/dt$) plot is a straight line in its later portion (60–100% U). The λ_{cH} and ultimate

primary settlement δ_{100} can be evaluated from the slope of the δ –($d\delta/dt$) plot without knowing the time and settlement at the instant of load increment. In secondary consolidation, the δ – $\log(d\delta/dt)$ plot is a straight line having the same slope as in the case of the δ – $\log(t)$ plot, and the coefficient of secondary consolidation can be evaluated as in the case of the δ – $\log(t)$ plot. The entire δ – t data of present, past and future can be generated by looking at the trend/rate of primary/secondary settlement over some time (Tewatia, 2013a).

However, for a usual case that can be expected in permeating rock, the clay drains outwards only, and Equation 49 can be written as

$$U = 1 - \frac{8}{\pi^2} \sum_{N=0}^{\infty} \frac{1}{(2N+1)^2} \epsilon^{-\frac{(2N+1)^2 \pi^2}{4} \lambda_{cH} t} \quad (67)$$

Comparing it with 1D Equation 68 from Terzaghi (1923),

$$U = 1 - \frac{8}{\pi^2} \sum_{N=0}^{\infty} \frac{1}{(2N+1)^2} \epsilon^{-\frac{(2N+1)^2 \pi^2}{4} T} \quad (68)$$

it is seen that

$$69. \quad T = \lambda_{cH} t$$

Therefore, it can be deduced that in usual cases of 3D consolidation of clayey gouge (of complicated shape, drainage, loadings, etc.) consolidating amid permeating rock, the shapes of settlement against rate of settlement plots in various formats are the same as in the case of 1D consolidation (Figure 7), only c_v/H^2 is increased by λ times. These are discussed at length by Tewatia (1998, 2013a) and Tewatia *et al.* (2007). However, the properties of these plots that are useful in this discussion are summarised here.

The δ –($dt/d\delta$) plot is a straight line in the initial portion (0–40% U), and its intercept on δ axis gives δ_0 . The δ –($d\delta/dt$) plot is a straight line in the later portion (60–100% U), and its intercept on δ axis gives δ_{100} . The λ_{cH} can be evaluated from Equation 52. On the

Figure 7. Shape of $U\text{-}\log_{10}(dU/dT)$ or settlement against rate of settlement (semi-log) plot

$\delta\text{-}\log_{10}(d\delta/dt)$ plot (S-curve, Figure 7), the initial portion is convex upwards and the latter portion is convex downwards. The slope of the S-curve in the initial portion is

$$70. \quad S_U = (\ln 10)(\delta_U - \delta_0)$$

The slope of the S-curve in the latter portion is

$$71. \quad S_U = (\ln 10)(\delta_{100} - \delta_U)$$

The slope of tangent at the point of inflection ($U = 50\%$) gives the amount of settlement

$$72. \quad S_{50} = \delta_{100} - \delta_0$$

By little experience, just by looking at a small portion of the S-curve in the field, an engineer can tell the approximate U without any calculations. The exact U can be found with the help of the equations given above.

Authors' reply

Ever since the inception of early investigations, the consolidation of clays has been a kernel of keen controversies (de Boer, 2005) and

distinguished discussions (Davis and Raymond, 1965). The authors appreciate the interest of the discussers in the consolidation of clayey gouge amid permeating rock masses. The discussers have drawn some useful insights to the consolidation of clayey gouge through a set of arguments, observations and published work. The authors also take a conscious note of work on consolidation reported by the discussers. The discussers have referred to a general theoretical equation for drainage. They have reported that the consolidation of clayey gouge amid permeating rock masses in geo-environment is always in progress and settlement, time and pressure at the beginning of consolidation (or at the instant of load increment) are not well known. The discussers assume that the pressure on clay in the beginning might have been increasing non-uniformly for an unknown duration, consolidating, with complicated shape, size, drainage conditions and set stages for primary and secondary consolidation. They also have referred to a method for determining in situ stress in rock/clay, using time and settlement plots since the beginning of consolidation.

In response to the discussers' contribution, the authors propose to explain their viewpoint on the consolidation of clayey gouge frequently occurring amid permeating rock masses as follows.

Exact and approximate solutions

The authors have offered an exact numerical solution using Fourier transform to estimate consolidation, with propositions of Dirichlet's problem as shown by the central line in Figure 8(a) with frequently encountered 2D drainage conditions and a rate of diffusion (C^2) described in space (consisting of orthogonal directions namely x and z) and time (t) for pore pressure (u).

$$73. \quad \frac{\partial u}{\partial t} = C^2 \nabla^2 u$$

$$\text{where, } \nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}$$

Most of the solutions for 2D and 3D drainage so far have referred to this quantity as corresponding to some experimental observations of ($C^2 = C_v$) in 1D oedometer, while the authors have considered an exact numerical solution based upon 2D drainage. Furthermore, most investigators, including the discussers, used a multiplier to time factor in a deviation in the consolidation estimate from the formulation of a 1D expression, while the authors have considered a multiplier to time factor in an exact numerical solution based upon 2D drainage.

Permeability as a tensor

The approximate solution normally captured the observed variation in the 1D, U against T_v plots using some kind of modifiers for the time factor, say, ' r ' by the authors, λ by the discussers and other variants prevalent in the literature (Di Francesco, 2011), conceiving 0th order (by Terzaghi, 1925), first order (by authors) or second

Figure 8. (a) Exact solution to the 1D, 2D and 3D consolidation (with varied initial convexities and terminal trajectories) at $T_v = 0.2$, the degree of consolidation has been approximated as $U_{1D} = 50\%$, $U_{2D} = 76\%$ and $U_{3D} = 88\%$. (b) Trajectory of primary

consolidation (full lines) in relation to the initial consolidation in exact solution (dotted lines) to the 1D, 2D and 3D consolidation with reference to initial convexities.

order tensorial variations in permeability (Bai *et al.*, 1999; de Boer, 2000; Kubik, 1979; Kubik and Sawczuk, 1983; Liakopoulos, 1965; Oda, 1985) and deformability in the mass materials and rock masses and hence in the coefficient of consolidation ($C^2 = C_v$). As such, the values of surface permeability (as scalar) amid rock masses may also be experimentally obtained and simulated (Singh *et al.*, 2013).

The authors have provided the values of ' r ' with reference to the convexities in the exact solution for the 2D case extended to 1D and 3D problems [see Tables 13 to 17 in the context of the exact and approximate solutions from 1D to 2D and 3D solutions and from 2D to 1D and 3D (as variant $r \rightarrow 0, 1$ and 2)]. Presumably, the discussers have also conceived the values of λ with reference to the

Drainage
condition

Exact solution for U_{av}

1D^a

$$1 - \sum_{k=0}^{\infty} \frac{8}{m^2 \pi^2} \exp \left[-T_v \frac{m^2 \pi^2}{4} \right]$$

2D^b

$$1 - \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \frac{64}{m^2 n^2 (\pi^2)^2} \exp \left[-T_v \left(\frac{m^2 \pi^2}{4} + r \frac{n^2 \pi^2}{4} \right) \right]$$

3D^c

$$1 - \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \sum_{j=0}^{\infty} \frac{512}{m^2 n^2 p^2 (\pi^2)^3} \exp \left[-T_v \left(\frac{m^2 \pi^2}{4} + rx \frac{n^2 \pi^2}{4} + ry \frac{p^2 \pi^2}{4} \right) \right]$$

where $m = 2k + 1$, $n = 2l + 1$, $p = 2j + 1$, while $k, l, j \in N$
for exact 2D case, $r = 1$ and for exact 3D case, $r_x = r_y = 1$.

^a Terzaghi (1925).

^{b, c} Authors' solution.

Table 13. Exact solution for consolidation of clayey gouge for varying drainage conditions

Dominant drainage condition	Frequency of occurrence amid rock masses	$r = rx, ry^a$	r^a	λ^b	C_v/H_z^2 vs $C_h/L_x^2; C_h/L_y^2$
1D	Less frequent	$rx = ry = 0$	$r = 0$	1	$C_v/H_z^2 \gg C_h/L_x^2; C_h/L_y^2$
2D	More frequent	$rx = r$ $ry = 0$ or $ry = r$ $rx = 0$	$r = 1$	$1 + rx$ or $1 + ry$	$C_h/L_x^2 \gg C_h/L_y^2$ or $C_h/L_y^2 \gg C_h/L_x^2$
3D	Less frequent	$rx = ry = 1$ $2Hz = 2Lx,$ $2Lx = 2Ly.$	$r = 2$	$1 + rx + ry$	$C_v/H_z^2 = C_h/L_x^2 = C_h/L_y^2$

$$r_x = \frac{C_{hx} (2Hz)^2}{C_{vz} (2Lx)^2}, r_y = \frac{C_{hy} (2Hz)^2}{C_{vz} (2Ly)^2}$$

^a Authors' proposal, ^b Discussers' proposal.

Table 14. Comparison of rock mass characteristic parameters for approximate solution to consolidation of clayey gouge amid permeating rock masses

Drainage condition	Differential equation	Self-similar solutions
1D	$C_z \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t}$	$U_{av} = 1 - (1 - Uz)^a$
2D	$C_x \frac{\partial^2 u}{\partial x^2} + C_z \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t}$	$U_{av} = 1 - (1 - Ux)(1 - Uz)^{b, c}$
3D	$C_x \frac{\partial^2 u}{\partial x^2} + C_y \frac{\partial^2 u}{\partial y^2} + C_z \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t}$	$U_{av} = 1 - (1 - Ux)(1 - Uy)(1 - Uz)^c$

^aTerzaghi (1925), ^bScott (1963), ^cNewman (1931).

Table 15. Initial valued problem and self-similar solutions for consolidation of clayey gouge

Drainage condition	Exact solution for U_{av}^a	Exact solution for U_{av}^b
1D	$1 - \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \frac{64}{(mn\pi^2)^2} \exp \left[-T_v \frac{m^2 \pi^2}{4} \right]$	$1 - \sum_{k=0}^{\infty} \frac{8}{m^2 \pi^2} \exp \left[-T_v \frac{m^2 \pi^2}{4} \right]$
2D	$1 - \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \frac{64}{(mn\pi^2)^2} \exp \left[-T_v \left(r \frac{n^2 \pi^2}{4} + \frac{m^2 \pi^2}{4} \right) \right]$	—

^aAuthors' solution based upon Dirichlet's problem.

^bTerzaghi (1925).

Table 16. Solution for consolidation of clayey gouge amid permeating rock masses with varied initial convexities

Drainage condition	Exact solution for U_{av}^a	Solution for U_{av}^b
1D	$1 - \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \frac{64}{(mn\pi^2)^2} \exp \left[-T_v \left(\frac{n^2 \pi^2}{4} \right) \right]$	$1 - \sum_{k=0}^{\infty} \frac{8}{m^2 \pi^2} \exp \left[-T_v \frac{m^2 \pi^2}{4} \right]$
2D	$1 - \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \frac{64}{(mn\pi^2)^2} \exp \left[-T_v \left(rx \frac{m^2 \pi^2}{4} + \frac{n^2 \pi^2}{4} \right) \right]$	$1 - \sum_{k=0}^{\infty} \frac{8}{m^2 \pi^2} \exp \left[-T_v (1 + rx) \frac{m^2 \pi^2}{4} \right]$
3D	$1 - \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \frac{64}{(mn\pi^2)^2} \exp \left[-T_v \left(rx \frac{m^2 \pi^2}{4} + ry \frac{m^2 \pi^2}{4} + \frac{n^2 \pi^2}{4} \right) \right]$	$1 - \sum_{k=0}^{\infty} \frac{8}{m^2 \pi^2} \exp \left[-T_v (1 + rx + ry) \frac{m^2 \pi^2}{4} \right]$

^a Authors' solution based upon 2D convexities.

^b Authors' approximation based upon 1D convexities.

^b Discussers' approximation as $\lambda = 1 + rx + ry$.

Table 17. Solution for consolidation of clayey gouge with convexities of 1D and 2D drainage

convexities in exact solution as the 1D case extended to 2D and 3D problems.

It is understood that the initial consolidation (initial separations that are overtly unknown as has been rightly pointed out by the discussers), in fact, determines the terminal trajectory of the U

against T_v plots (Figures 8(a) and 8(b)). The isometric distribution of U in x - z space at $T_v = 0.2$ for $r = 0$ and 1 is shown in Figures 9(a) and 9(b), respectively.

Since anisotropy is distributive in space as per Dirichlet's problem, the authors have proposed approximate expressions based upon

Figure 9. (a) Isometric representation of U (at $r = 0$ and $T_v = 0.2$) from exact 2D solution as $U_{1D} = 50\%$. (b) Isometric representation of U (at $r = 1$ and $T_v = 0.2$) from exact 2D solution as $U_{2D} = 76\%$.

exact solutions to a variety of drainage situations and shapes captured by a parameter ($r = 0, 1$ and 2 as the order of tensor) estimated from rock mass characteristics, drainage features and extendible to 1D as well as 3D drainage conditions for gouge amid permeating rock masses. Instead of the parameter (r), the discussers use another parameter (λ) to describe 1D to 3D conditions, which can be numerically related to ' r ' as shown in Table 14. A wide-ranging approximation of primary consolidation U_p can be made for varying values of ' r ' as shown in Figure 10.

Deformability and estimating stresses amid permeating rock masses

The coefficient of consolidation is a product of two tensorial quantities, namely permeability (Bai *et al.*, 1999; Kubik, 1979; Liakopoulos, 1965; Oda, 1985) per unit density and deformability (Trivedi, 2013; Zhang, 2010), which are functions of fracture density, joint and gouge parameters included in J_{fg} , and a hardening parameter C_h . J_{fg} is a modified joint factor that considers the occurrence of varied engineering possibilities amid rock masses in the range of consolidation pressure. Because of compression of the gouge, the rock mass can undergo significant volume change (Trivedi, 2010) with drainage and shear. The dependence of deformability on J_{fg} and a hardening parameter C_h is shown in

Figures 11(a) and 11(b) using some field and numerical data. The average deformability of rock masses is obtained using joint and gouge parameters included in J_{fg} , and a hardening parameter C_h . E_{mr} is a ratio of deformation modulus of rock mass to the intact rock such that $E_{mr} = \exp(C_h J_{fg})$. The hardening parameter evolves from the plasticity parameter and initial isotropic pressure. The hardening parameter depends upon plasticity parameter α_p and initial isotropic pressure such that $C_h = \kappa[(p_i/\sigma_c)^\eta]$. The values of the constants κ and η depend upon α_p (Trivedi, 2013).

The in situ geo-technical explorations allow us to capture some values of E_{mr} and J_{fg} . Using values of E_{mr} and J_{fg} in Figure 11(a), a value of C_h can be found. Figure 11(a) shows estimates of deformation modulus of rock masses interpreted from the field data of Chun *et al.* (2009). The geo-technical investigations also provide us with information to interpret a plasticity parameter α_p . Using values of C_h and α_p in Figure 11(b), a value of initial isotropic pressure amid rock masses can be obtained.

Under the varied pressure conditions of gouge amid rock masses as described by the authors, the deformation and velocity fields (namely, $\delta\text{-}\sigma'$ against $\delta\text{-}d\sigma'/d\delta$ and U against $\log_{10}(dU/dT)$, respectively) do not remain coaxial, hence precise determination of in situ stresses may remain elusive by the said technique as proposed by the discussers.

Figure 10. Trajectory of primary consolidation ($r = 0\text{--}10$) in approximate solution (derived from the exact 2D solution) in relation to the 1D (as $r = 0$), 2D (as $r = 1$) and 3D (as $r = 2$) consolidation and modified initial convexities

Figure 11. (a) Estimation of deformability of rock masses using joint and gouge parameters included in J_{fg} , and a hardening parameter C_h . E_{mr} represents a ratio of deformation modulus of rock mass to the intact rock. The hardening parameter depends upon the plasticity parameter and initial isotropic pressure such that $E_{mr} = \exp(C_h J_{fg})$. (b) Relationship of the hardening parameter C_h and initial isotropic pressure ratio; the hardening parameter depends upon plasticity parameter α_p and initial isotropic pressure.

The hardening parameter depends upon initial plasticity and initial isotropic pressure such that $C_h = \kappa[(p/\sigma_r)^\eta]$. The values of the constants κ and η depend upon α_p . p/σ_r is a ratio of initial isotropic pressure and uniaxial compressive strength of intact rock. The plasticity parameter is such that $\sigma_{mr} = \exp(\alpha_p J_{fg}^p)$. J_{fg}^p is a modified joint factor at plastic failure. σ_{mr} represents a ratio of compressive strength of rock mass to the intact rock.

Nevertheless, the discussers have made an interesting contribution with reference to few presumably coaxial experimental observations; hence, there is a need to explore deformation and velocity fields of consolidating clayey gouge amid permeating rock masses for varied coaxial and non-coaxial deformations.

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